Solutions Of Hatcher Algebraic Topology Exercise 4

Introduction to Topological ManifoldsSpringer Science & Business Media

Annotation. The book is intended as a text for a two-semester course in topology and algebraic topology at the advanced undergraduate orbeginning graduate level. There are over 500 exercises, 114 figures, numerous diagrams. The general direction of the book is towardhomotopy theory with a geometric point of view. This book would providea more than adequate background for a standard algebraic topology coursethat begins with homology theory. For more information seewww.bangor.ac.uk/r.brown/topgpds.htmlThis version dated April 19, 2006, has a number of corrections made.

Building on rudimentary knowledge of real analysis, point-set topology, and basic algebra, Basic Algebraic Topology provides plenty of material for a two-semester course in algebraic topology. The book first introduces the necessary fundamental concepts, such as relative homotopy, fibrations and cofibrations, category theory, cell complexes, and si

This book brings the most important aspects of modern topology within reach of a second-year undergraduate student. It successfully unites the most exciting aspects of modern topology with those that are most useful for research, leaving readers prepared and motivated for further study. Written from a thoroughly modern perspective, every topic is introduced with an explanation of why it is being studied, and a huge number of examples provide further motivation. The book is ideal for self-study and assumes only a familiarity with the notion of continuity and basic algebra.

(Cartan sub Lie algebra, roots, Weyl group, Dynkin diagram, . . .) and the classification, as found by Killing and Cartan (the list of all semisimple Lie algebras consists of (1) the special- linear ones, i. e. all matrices (of any fixed dimension) with trace 0, (2) the orthogonal ones, i. e. all skewsymmetric ma trices (of any fixed dimension), (3) the symplectic ones, i. e. all matrices M (of any fixed even dimension) that satisfy M J = - J MT with a certain non-degenerate skewsymmetric matrix J, and (4) five special Lie algebras G2, F , E , E , E , of dimensions 14,52,78,133,248, the "exceptional Lie 4 6 7 s algebras" , that just somehow appear in the process). There is also a discus sion of the compact form and other real forms of a (complex) semisimple Lie algebra, and a section on automorphisms. The third chapter brings the theory of the finite dimensional representations of a semisimple Lie alge bra, with the highest or extreme weight as central notion. The proof for the existence of representations is an ad hoc version of the present standard proof, but avoids explicit use of the Poincare-Birkhoff-Witt theorem. Complete reducibility is proved, as usual, with J. H. C. Whitehead's proof (the first proof, by H. Weyl, was analytical-topological and used the exis tence of a compact form of the group in question). Then come H.

The book offers a good introduction to topology through solved exercises. It is mainly intended for undergraduate students. Most exercises are given with detailed solutions. In the second edition, some significant changes have been made, other than the additional exercises. There are also additional proofs (as exercises) of many results in the old section "What You Need To Know", which has been improved and renamed in the new edition as "Essential Background". Indeed, it has been considerably beefed up as it now includes more remarks and results for readers' convenience. The interesting sections "True or False" and "Tests" have remained as they were, apart from a very few changes.

Since the early 1980s, there has been an explosive growth in 4-manifold theory, particularly due to the influx of interest and ideas from gauge theory and algebraic geometry. This book offers an exposition of the subject from the topological point of view. It bridges the gap to other disciplines and presents classical but important topological techniques that have not previously appeared in the literature. Part I of the text presents the basics of the theory at the second-year graduate level and offers an overview of current research. Part II is devoted to an exposition of Kirby calculus, or handlebody theory on 4-manifolds. It is both elementary and comprehensive. Part III offers in-depth treatments of a broad range of topics from current 4-manifold research. Topics include branched coverings and the geography of complex surfaces, elliptic and Lefschetz fibrations, h-cobordisms, symplectic 4-manifolds, and Stein surfaces. The authors present many important applications. The text is supplemented with over 300 illustrations and numerous exercises, with solutions given in the book. I greatly recommend this wonderful book to any researcher in 4-manifold topology for the novel ideas, techniques, constructions, and computations on the topic, presented in a very fascinating way. I think really that every student, mathematician, and researcher interested in 4-manifold topology, should own a copy of this beautiful book. --Zentralblatt MATH This book gives an excellent introduction into the theory of 4-manifolds and can be strongly recommended to beginners in this field ... carefully and clearly written; the authors have evidently paid great attention to the presentation of the material ... contains many really pretty and interesting examples and a great number of exercises; the final chapter is then devoted to solutions of some of these ... this type of presentation makes the subject more attractive and its study easier. --European Mathematical Society Newsletter

Simplicial sets are discrete analogs of topological spaces. They have played a central role in algebraic topology ever since their introduction in the late 1940s, and they also play an important role in other areas such as geometric topology and algebraic geometry. On a formal level, the homotopy theory of simplicial sets is equivalent to the homotopy theory of topological spaces. In view of this equivalence, one can apply discrete, algebraic techniques to perform basic topological constructions. These techniques are particularly appropriate in the theory of localization and completion of topological spaces, which was developed in the early 1970s. Since it was first published in 1967, Simplicial Objects in Algebraic Topology has been the standard reference for the theory of simplicial sets and their relationship to the homotopy theory of topological spaces. J. Peter May gives a lucid account of the basic homotopy theory of simplicial sets, together with the equivalence of homotopy theories alluded to above. The central theme is the simplicial approach to the theory of fibrations and bundles, and especially the algebraization of fibration and bundle theory in terms of "twisted Cartesian products." The Serre spectral sequence is described in terms of this algebraization. Other topics treated in detail include Eilenberg-MacLane complexes, Postnikov systems, simplicial groups, classifying complexes, simplicial Abelian groups, and acyclic models. "Simplicial Objects in Algebraic Topology presents much of the elementary material of algebraic topology from the semi-simplicial viewpoint. It should prove very valuable to anyone wishing to learn semi-simplicial topology. [May] has included detailed proofs, and he has succeeded very well in the task of organizing a large body of previously scattered material."—Mathematical Review

With firm foundations dating only from the 1950s, algebraic topology is a relatively young area of mathematics. There are very few textbooks that treat fundamental topics beyond a first course, and many topics now essential to the field are not treated in any textbook. J. Peter May’s A Concise Course in Algebraic Topology addresses the standard first course material, such as fundamental groups, covering spaces, the basics of homotopy theory, and homology and cohomology. In this sequel, May and his coauthor, Kathleen Ponto, cover topics that are essential for algebraic topologists and others interested in algebraic topology, but that are not treated in standard texts. They focus on the localization and completion of topological spaces, model categories, and Hopf algebras. The first half of the book sets out the basic theory of localization and completion of nilpotent spaces, using the most elementary treatment the authors know of. It makes no use of simplicial techniques or model categories, and it provides full details of other necessary preliminaries. With these topics as motivation, most of the second half of the book sets out the theory of model categories, which is the central organizing framework for homotopical algebra in general. Examples from topology and homological algebra are treated in parallel. A short last part develops the basic theory of bialgebras and Hopf algebras.

This text explains nontrivial applications of metric space topology to analysis. Covers metric space, point-set topology, and algebraic topology. Includes exercises, selected answers, and 51 illustrations. 1983 edition.

This book provides an accessible introduction to algebraic topology, a field at the intersection of topology, geometry and algebra, together with its applications. Moreover, it covers several related topics that are in fact important in the overall scheme of algebraic topology. Comprising eighteen chapters and two appendices, the book integrates various concepts of algebraic topology, supported by examples, exercises, applications and historical notes. Primarily intended as a textbook, the book offers a valuable resource for undergraduate, postgraduate and advanced mathematics students alike. Focusing more on the geometric than on algebraic aspects of the subject, as well as its natural development, the book conveys the basic language of modern algebraic topology by exploring homotopy, homology and cohomology theories, and examines a variety of spaces: spheres, projective spaces, classical groups and their quotient spaces, function spaces, polyhedra, topological groups, Lie groups and cell complexes, etc. The book studies a variety of maps, which are continuous functions between spaces. It also reveals the importance of algebraic topology in contemporary mathematics, theoretical physics, computer science, chemistry, economics, and the biological and medical sciences, and encourages students to engage in further study.

This book surveys the fundamental ideas of algebraic topology. The first part covers the fundamental group, its definition and application in the study of covering spaces. The second part turns to homology theory including cohomology, cup products, cohomology operations and topological manifolds. The final part is devoted to Homotropy theory, including basic facts about homotropy groups and applications to obstruction theory.

Algebraic topology is the study of the global properties of spaces by means of algebra. It is an important branch of modern

mathematics with a wide degree of applicability to other fields, including geometric topology, differential geometry, functional analysis, differential equations, algebraic geometry, number theory, and theoretical physics. This book provides an introduction to the basic concepts and methods of algebraic topology for the beginner. It presents elements of both homology theory and homotopy theory, and includes various applications. The author's intention is to rely on the geometric approach by appealing to the reader's own intuition to help understanding. The numerous illustrations in the text also serve this purpose. Two features make the text different from the standard literature: first, special attention is given to providing explicit algorithms for calculating the homology groups and for manipulating the fundamental groups. Second, the book contains many exercises, all of which are supplied with hints or solutions. This makes the book suitable for both classroom use and for independent study.
This textbook is intended for a course in algebraic topology at the beginning graduate level. The main topics covered are the classification of compact 2-manifolds, the fundamental group, covering spaces, singular homology theory, and singular cohomology theory. These topics are developed systematically, avoiding all unnecessary definitions, terminology, and technical machinery. The text consists of material from the first five chapters of the author's earlier book, Algebraic Topology; an Introduction (GTM 56) together with almost all of his book, Singular Homology Theory (GTM 70). The material from the two earlier books has been substantially revised, corrected, and brought up to date.
Manifolds, the higher-dimensional analogs of smooth curves and surfaces, are fundamental objects in modern mathematics. Combining aspects of algebra, topology, and analysis, manifolds have also been applied to classical mechanics, general relativity, and quantum field theory. In this streamlined introduction to the subject, the theory of manifolds is presented with the aim of helping the reader achieve a rapid mastery of the essential topics. By the end of the book the reader should be able to compute, at least for simple spaces, one of the most basic topological invariants of a manifold, its de Rham cohomology. Along the way, the reader acquires the knowledge and skills necessary for further study of geometry and topology. The requisite point-set topology is included in an appendix of twenty pages; other appendices review facts from real analysis and linear algebra. Hints and solutions are provided to many of the exercises and problems. This work may be used as the text for a one-semester graduate or advanced undergraduate course, as well as by students engaged in self-study. Requiring only minimal undergraduate prerequisites, 'Introduction to Manifolds' is also an excellent foundation for Springer's GTM 82, 'Differential Forms in Algebraic Topology'.
Manifolds play an important role in topology, geometry, complex analysis, algebra, and classical mechanics. Learning manifolds differs from most other introductory mathematics in that the subject matter is often completely unfamiliar. This introduction guides readers by explaining the roles manifolds play in diverse branches of mathematics and physics. The book begins with the basics of general topology and gently moves to manifolds, the fundamental group, and covering spaces.
The amount of algebraic topology a graduate student specializing in topology must learn can be intimidating. Moreover, by their second year of graduate studies, students must make the transition from understanding simple proofs line-by-line to understanding the overall structure of proofs of difficult theorems. To help students make this transition, the material in

this book is presented in an increasingly sophisticated manner. It is intended to bridge the gap between algebraic and geometric topology, both by providing the algebraic tools that a geometric topologist needs and by concentrating on those areas of algebraic topology that are geometrically motivated. Prerequisites for using this book include basic set-theoretic topology, the definition of CW-complexes, some knowledge of the fundamental group/covering space theory, and the construction of singular homology. Most of this material is briefly reviewed at the beginning of the book. The topics discussed by the authors include typical material for first- and second-year graduate courses. The core of the exposition consists of chapters on homotopy groups and on spectral sequences. There is also material that would interest students of geometric topology (homology with local coefficients and obstruction theory) and algebraic topology (spectra and generalized homology), as well as preparation for more advanced topics such as algebraic K-theory and the s-cobordism theorem. A unique feature of the book is the inclusion, at the end of each chapter, of several projects that require students to present proofs of substantial theorems and to write notes accompanying their explanations. Working on these projects allows students to grapple with the ``big picture'', teaches them how to give mathematical lectures, and prepares them for participating in research seminars. The book is designed as a textbook for graduate students studying algebraic and geometric topology and homotopy theory. It will also be useful for students from other fields such as differential geometry, algebraic geometry, and homological algebra. The exposition in the text is clear; special cases are presented over complex general statements.

Informally, K-theory is a tool for probing the structure of a mathematical object such as a ring or a topological space in terms of suitably parameterized vector spaces and producing important intrinsic invariants which are useful in the study of algebr

This book is written as a textbook on algebraic topology. The first part covers the material for two introductory courses about homotopy and homology. The second part presents more advanced applications and concepts (duality, characteristic classes, homotopy groups of spheres, bordism). The author recommends starting an introductory course with homotopy theory. For this purpose, classical results are presented with new elementary proofs. Alternatively, one could start more traditionally with singular and axiomatic homology. Additional chapters are devoted to the geometry of manifolds, cell complexes and fibre bundles. A special feature is the rich supply of nearly 500 exercises and problems. Several sections include topics which have not appeared before in textbooks as well as simplified proofs for some important results. Prerequisites are standard point set topology (as recalled in the first chapter), elementary algebraic notions (modules, tensor product), and some terminology from category theory. The aim of the book is to introduce advanced undergraduate and graduate (master's) students to basic tools, concepts and results of algebraic topology.

Sufficient background material from geometry and algebra is included.
This volume deals with the theory of finite topological spaces and its relationship with the homotopy and simple homotopy theory of polyhedra. The interaction between their intrinsic combinatorial and topological structures makes finite spaces a useful tool for studying problems in Topology, Algebra and Geometry from a new perspective. In particular, the methods developed in this manuscript are used to study Quillen's conjecture on the poset of p-subgroups of a finite group and the Andrews-Curtis conjecture on the 3-deformability of contractible two-dimensional complexes. This self-contained work constitutes the first detailed exposition on the algebraic topology of finite spaces. It is intended for topologists and combinatorialists, but it is also recommended for advanced undergraduate students and graduate students with a modest knowledge of Algebraic Topology.
Algebraic topology is a basic part of modern mathematics, and some knowledge of this area is indispensable for any advanced work relating to geometry, including topology itself, differential geometry, algebraic geometry, and Lie groups. This book provides a detailed treatment of algebraic topology both for teachers of the subject and for advanced graduate students in mathematics either specializing in this area or continuing on to other fields. J. Peter May's approach reflects the enormous internal developments within algebraic topology over the past several decades, most of which are largely unknown to mathematicians in other fields. But he also retains the classical presentations of various topics where appropriate. Most chapters end with problems that further explore and refine the concepts presented. The final four chapters provide sketches of substantial areas of algebraic topology that are normally omitted from introductory texts, and the book concludes with a list of suggested readings for those interested in delving further into the field.
This book is an exposition of the theoretical foundations of hyperbolic manifolds. It is intended to be used both as a textbook and as a reference. Particular emphasis has been placed on readability and completeness of ar gument. The treatment of the material is for the most part elementary and self-contained. The reader is assumed to have a basic knowledge of algebra and topology at the first-year graduate level of an American university. The book is divided into three parts. The first part, consisting of Chap ters 1-7, is concerned with hyperbolic geometry and basic properties of discrete groups of isometries of hyperbolic space. The main results are the existence theorem for discrete reflection groups, the Bieberbach theorems, and Selberg's lemma. The second part, consisting of Chapters 8-12, is de voted to the theory of hyperbolic manifolds. The main results are Mostow's rigidity theorem and the determination of the structure of geometrically finite hyperbolic manifolds. The third part, consisting of Chapter 13, in tegrates the first two parts in a development of the theory of hyperbolic orbifolds. The main results are the construction of the universal orbifold covering space and Poincare's fundamental polyhedron theorem.

The theory of characteristic classes provides a meeting ground for the various disciplines of differential topology, differential and algebraic geometry, cohomology, and fiber bundle theory. As such, it is a fundamental and an essential tool in the study of differentiable manifolds. In this volume, the authors provide a thorough introduction to characteristic classes, with detailed studies of Stiefel-Whitney classes, Chern classes, Pontrjagin classes, and the Euler class. Three appendices cover the basics of cohomology theory and the differential forms approach to characteristic classes, and provide an account of Bernoulli numbers. Based on lecture notes of John Milnor, which first appeared at Princeton University in 1957 and have been widely studied by graduate students of topology ever since, this published version has been completely revised and corrected.

Algebraic Topology and basic homotopy theory form a fundamental building block for much of modern mathematics. These lecture notes represent a culmination of many years of leading a two-semester course in this subject at MIT. The style is engaging and student-friendly, but precise. Every lecture is accompanied by exercises. It begins slowly in order to gather up students with a variety of backgrounds, but gains pace as the course progresses, and by the end the student has a command of all the basic techniques of classical homotopy theory.

Developed from a first-year graduate course in algebraic topology, this text is an informal introduction to some of the main ideas of contemporary homotopy and cohomology theory. The materials are structured around four core areas: de Rham theory, the Cech-de Rham complex, spectral sequences, and characteristic classes. By using the de Rham theory of differential forms as a prototype of cohomology, the machineries of algebraic topology are made easier to assimilate. With its stress on concreteness, motivation, and readability, this book is equally suitable for self-study and as a one-semester course in topology.

In recent years, many students have been introduced to topology in high school mathematics. Having met the Mobius band, the seven bridges of Konigsberg, Euler's polyhedron formula, and knots, the student is led to expect that these picturesque ideas will come to full flower in university topology courses. What a disappointment "undergraduate topology" proves to be! In most institutions it is either a service course for analysts, on abstract spaces, or else an introduction to homological algebra in which the only geometric activity is the completion of commutative diagrams. Pictures are kept to a minimum, and at the end the student still does nr~ understand the simplest topological facts, such as the rcason why knots exist. In my opinion, a well-balanced introduction to topology should stress its intuitive geometric aspect, while admitting the legitimate interest that analysts and algebraists have in the subject. At any rate, this is the aim of the present book. In support of this view, I have followed the historical development where practicable, since it clearly shows the influence of geometric thought at all stages. This is not to claim that topology received its main impetus from geometric recreations like the seven bridges; rather, it resulted from the l'isualization of problems from other parts of mathematics-complex analysis (Riemann), mechanics (Poincare), and group theory (Dehn). It is these connec tions to other parts of mathematics which make topology an important as well as a beautiful subject.

"Topology can present significant challenges for undergraduate students of mathematics and the sciences. 'Understanding topology' aims to change that. The perfect introductory topology textbook, 'Understanding topology' requires only a knowledge of calculus and a general familiarity with set theory and logic. Equally approachable and rigorous, the book's clear organization, worked examples, and concise writing style support a thorough understanding of basic topological principles. Professor Shaun V. Ault's unique emphasis on fascinating applications, from chemical dynamics to determining the shape of the universe, will engage students in a way traditional topology textbooks do not"--Back cover.

This text contains a detailed introduction to general topology and an introduction to algebraic topology via its most classical and elementary segment. Proofs of theorems are separated from their formulations and are gathered at the end of each chapter, making this book appear like a problem book and also giving it appeal to the expert as a handbook. The book includes about 1,000 exercises.

This self-contained introduction to algebraic topology is suitable for a number of topology courses. It consists of about one quarter 'general topology' (without its usual pathologies) and three quarters 'algebraic topology' (centred around the fundamental group, a readily grasped topic which gives a good idea of what algebraic topology is). The book has emerged from courses given at the University of Newcastle-upon-Tyne to senior undergraduates and beginning postgraduates. It has been written at a level which will enable the reader to use it for self-study as well as a course book. The approach is leisurely and a geometric flavour is evident throughout. The many illustrations and over 350 exercises will prove invaluable as a teaching aid. This account will be welcomed by advanced students of pure mathematics at colleges and universities.

Topology is a large subject with many branches broadly categorized as algebraic topology, point-set topology, and geometric topology. Point-set topology is the main language for a broad variety of mathematical disciplines. Algebraic topology serves as a powerful tool for studying the problems in geometry and numerous other areas of mathematics. Elements of Topology provides a basic introduction to point-set topology and algebraic topology. It is intended for advanced undergraduate and beginning graduate students with working knowledge of analysis and algebra. Topics discussed include the theory of convergence, function spaces, topological transformation groups, fundamental groups, and covering spaces. The author makes the subject accessible by providing more than 250 worked examples and counterexamples with applications. The text also includes numerous end-of-section exercises to put the material into context.

An introductory textbook suitable for use in a course or for self-study, featuring broad coverage of the subject and a readable exposition, with many examples and exercises.

This book gives an introduction to the mathematics and applications comprising the new field of applied topology. The elements of this subject are surveyed in the context of applications drawn from the biological, economic, engineering, physical, and statistical sciences.

In this broad introduction to topology, the author searches for topological invariants of spaces, together with techniques for their calculating. Students with knowledge of real analysis, elementary group theory, and linear algebra will quickly become familiar with a wide variety of techniques and applications involving point-set, geometric, and algebraic topology. Over 139 illustrations and more than 350 problems of various difficulties help students gain a thorough understanding of the subject.

Great first book on algebraic topology. Introduces (co)homology through singular theory.

Elements of Algebraic Topology provides the most concrete approach to the subject. With coverage of homology and cohomology theory, universal coefficient theorems, Kunneth theorem, duality in manifolds, and applications to classical theorems of point-set topology, this book is perfect for comunicating complex topics and the fun nature of algebraic topology for beginners.

The single most difficult thing one faces when one begins to learn a new branch of mathematics is to get a feel for the mathematical sense of the subject. The purpose of this book is to help the aspiring reader acquire this essential common sense about algebraic topology in a short period of time. To this end, Sato leads the reader through simple but meaningful examples in concrete terms. Moreover, results are not discussed in their greatest possible generality, but in terms of the simplest and most essential cases. In response to suggestions from readers of the original edition of this book, Sato has added an appendix of useful definitions and results on sets, general topology, groups and such. He has also provided references. Topics covered include fundamental notions such as homeomorphisms, homotopy equivalence, fundamental groups and higher homotopy groups, homology and cohomology, fiber bundles, spectral sequences and characteristic classes. Objects and examples considered in the text include the torus, the Mobius strip, the Klein bottle, closed surfaces, cell complexes and vector bundles.

This book offers an introductory course in algebraic topology. Starting with general topology, it discusses differentiable manifolds, cohomology, products and duality, the fundamental group, homology theory, and homotopy theory. From the reviews: "An interesting and original graduate text in topology and geometry...a good lecturer can use this text to create a fine course....A beginning graduate student can use this text to learn a great deal of mathematics."—-MATHEMATICAL REVIEWS

Excellent text covers vector fields, plane homology and the Jordan Curve Theorem, surfaces, homology of complexes, more. Problems and exercises. Some knowledge of differential equations and multivariate calculus required.Bibliography. 1979 edition.

To the Teacher. This book is designed to introduce a student to some of the important ideas of algebraic topology by emphasizing the re lations of these ideas with other areas of mathematics. Rather than choosing one point of view of modem topology (homotopy theory, simplicial complexes, singular theory, axiomatic homology, differ ential topology, etc.), we concentrate our attention on concrete prob lems in low dimensions, introducing only as much algebraic machin ery as necessary for the problems we meet. This makes it possible to see a wider variety of important features of the subject than is usual in a beginning text. The book is designed for students of mathematics or science who are not aiming to become practicing algebraic topol ogists-without, we hope, discouraging budding topologists. We also feel that this approach is in better harmony with the historical devel opment of the subject. What would we like a student to know after a first course in to pology (assuming we reject the answer: half of what one would like the student to know after a second course in topology)? Our answers to this have guided the choice of material, which includes: under standing the relation between homology and integration, first on plane domains, later on Riemann surfaces and in higher dimensions; wind ing numbers and degrees of mappings, fixed-point theorems; appli cations such as the Jordan curve theorem, invariance of domain; in dices of vector fields and Euler characteristics; fundamental groups