Numerical Analysis Of Spectral Methods Theory And Applications Cbms Nsf Regional Conference Series In Applied Mathematics

This book is a pedagogical presentation of the application of spectral and pseudospectral methods to kinetic theory and quantum mechanics. There are additional applications to astrophysics, engineering, biology and many other fields. The main objective of this book is to provide the basic concepts to enable the use of spectral and pseudospectral methods to solve problems in diverse fields of interest and to a wide audience. While spectral methods are generally based on Fourier Series or Chebychev polynomials, non-classical polynomials and associated quadratures are used for many of the applications presented in the book. Fourier series methods are summarized with a discussion of the resolution of the Gibbs phenomenon. Classical and non-classical quadratures are used for the evaluation of integrals in reaction dynamics including nuclear fusion, radial integrals in density functional theory, in elastic scattering theory and other applications. The subject matter includes the calculation of transport coefficients in gases and other gas dynamical problems based on spectral and pseudospectral solutions of the Boltzmann equation. Radiative transfer in astrophysics and atmospheric science, and applications to space physics are discussed. The relaxation of initial non-equilibrium distributions to equilibrium for several different systems is studied with the Boltzmann and Fokker-Planck equations. The eigenvalue spectra of the linear operators in the Boltzmann, Fokker-Planck and Schrödinger equations are studied with spectral and pseudospectral methods based on non-classical orthogonal polynomials. The numerical methods referred to as the Discrete Ordinate Method, Differential Quadrature, the Quadrature Discretization Method, the Discrete Variable Representation, the Lagrange Mesh Method, and others are discussed and compared. MATLAB codes are provided for most of the numerical results reported in the book - see Link under 'Additional Information' on the the right-hand column.

Spectral methods are well-suited to solve problems modeled by time-dependent partial differential equations: they are fast, efficient and accurate and widely used by mathematicians and practitioners. This class-tested 2007 introduction, the first on the subject, is ideal for graduate courses, or self-study. The authors describe the basic theory of spectral methods, allowing the reader to understand the techniques through numerous examples as well as more rigorous developments. They provide a detailed treatment of methods based on Fourier expansions and orthogonal polynomials (including discussions of stability, boundary conditions, filtering, and the extension from the linear to the nonlinear situation). Computational solution techniques for integration in time are dealt with by Runge-Kutta type methods. Several chapters are devoted to material not previously covered in book form, including stability theory for polynomial methods, techniques for problems with discontinuous solutions, round-off errors and the formulation of spectral methods on general grids. These will be especially helpful for practitioners.

The purpose of the book is to provide a rigorous treatment of spectral analysis using Fourier-based techniques in the geosciences, specifically geodesy and geophysics that deal with global and regional spatial data. The basic motivation is to get students to think about spatial data in the corresponding spectral domain. As such, the book emphasizes spatial-frequency analysis for data on the plane and sphere. Most books in spectral analysis develop the topic for time signals and are oriented to electrical and communications engineering. There are very few books for the spatial domain, which develop methods in two dimensions on the plane or the sphere for geodetic and geophysical applications. In the modern era of satellite remote sensing, these techniques are particularly important, as space-borne sensors generate global data (gravity, magnetics, topography, sea level, etc.). The proposed book includes applications to stochastic processes on the plane and sphere, which are important for many estimation problems that are based on some kind of stochastic constraints in inverse theory. There is a final chapter that briefly outlines wavelet analysis. Although a bit outside the scope, it is included to illustrate the contrast to traditional spectral analysis, going more toward a time-frequency analysis. For example, some applications such as wavelet-denoising, as opposed to low-pass filtering, are considered important supplements to the usual spectral methods. This book does not compete with the plethora of recent books on Spatial Data Analysis, which typically do not develop methods in the spectral domain. There are a few books on spectral methods in geophysics (none in geodesy) with which this book might compete. However, those books tend to emphasize specific applications, such as seismology or meteorology, rather than more general spatial signals on the plane and sphere. What this proposed book offers is more fundamental in this respect, and thus offers unique perspective for students in the geosciences.

This book explains how to solve partial differential equations numerically using single and multidomain spectral methods. It shows how only a few fundamental algorithms form the building blocks of any spectral code, even for problems with complex geometries.

This book deals with the application of spectral methods to problems of uncertainty propagation and quanti?cation in model-based computations. It speci?cally focuses on computational and algorithmic features of these methods which are most useful in dealing with models based on partial differential equations, with special att- tion to models arising in simulations of ?uid ?ows. Implementations are illustrated through applications to elementary problems, as well as more elaborate examples selected from the authors' interests in incompressible vortex-dominated ?ows and compressible ?ows at low Mach numbers. Spectral stochastic methods are probabilistic in nature, and are consequently rooted in the rich mathematical foundation associated with probability and measure spaces. Despite the authors' fascination with this foundation, the discussion only - ludes to those theoretical aspects needed to set the stage for subsequent applications. The book is authored by practitioners, and is primarily intended for researchers or graduate students in computational mathematics, physics, or ?uid dynamics. The book assumes familiarity with elementary methods for the numerical solution of time-dependent, partial differential equations; prior experience with spectral me- ods is naturally helpful though not essential. Full appreciation of elaborate examples in computational ?uid dynamics (CFD) would require familiarity with key, and in some cases delicate, features of the associated numerical methods. Besides these shortcomings, our aim is to treat algorithmic and computational aspects of spectral stochastic methods with details suf?cient to address and reconstruct all but those highly elaborate examples.

This book explains how, when and why the pseudospectral approach works.

A unified discussion of the formulation and analysis of special methods of mixed initial boundary-value problems. The focus is on the development of a new mathematical theory that explains why and how well spectral methods work. Included are interesting extensions of the classical numerical analysis.

A guide to the new research in the field of fractional order analysis Fractional Order Analysis contains the most recent research findings in fractional order analysis and its applications. The authors—noted experts on the topic—offer an examination of the theory, methods, applications, and the modern tools and techniques in the field of fractional order analysis. The information, tools, and applications presented can help develop mathematical methods and models with better accuracy. Comprehensive in scope, the book covers a range of topics including: new fractional operators, fractional derivatives, fractional differential equations, inequalities for different fractional derivatives and fractional integrals, fractional modeling related to transmission of Malaria, and dynamics of Zika virus with various fractional derivatives, and more. Designed to be an accessible text, several useful, relevant and connected topics can be found in one place, which is crucial for an understanding of the research problems of an applied nature. This book: Contains recent development in fractional calculus Offers a balance of theory, methods, and applications Puts the focus on fractional analysis and its interdisciplinary applications, such as fractional models for biological models Helps make research more relevant to real-life applications Written for researchers, professionals and practitioners, Fractional Order Analysis offers a comprehensive resource to fractional analysis and its many applications as well as information on the newest research.

This is a very lucid introduction to spectral methods emphasizing the mathematical aspects of the theory rather than the many applications in numerical analysis and the engineering sciences. The first part is a fairly complete introduction to Fourier series while the second emphasizes polynomial expansion methods like Chebyshev's. The author gives rigorous proofs of fundamental results related to one-dimensional advection and diffusions equations. The book addresses students as well as practitioners of numerical analysis.

Since the original publication of this book, available computer power has increased greatly. Today, scientific computing is playing an ever more prominent role as a tool in scientific discovery and engineering analysis. In this second edition, the key addition is an introduction to the finite element method. This is a widely used technique for solving partial differential equations (PDEs) in complex domains. This text introduces numerical methods and shows how to develop, analyse, and use them. Complete MATLAB programs for all the worked examples are now available at www.cambridge.org/Moin, and more than 30 exercises have been added. This thorough and practical book is intended as a first course in numerical analysis, primarily for new graduate students in engineering and physical science. Along with mastering the fundamentals of numerical methods, students will learn to write their own computer programs using standard numerical methods.

Since the publication of "Spectral Methods in Fluid Dynamics" 1988, spectral methods have become firmly established as a mainstream tool for scientific and engineering computation. The authors of that book have incorporated into this new edition the many improvements in the algorithms and the theory of spectral methods that have been made since then. This latest book retains the tight integration between the theoretical and practical aspects of spectral methods, and the chapters are enhanced with material on the Galerkin with numerical integration version of spectral methods. The discussion of direct and iterative solution methods is also greatly expanded.

Handbook of Numerical Methods for Hyperbolic Problems explores the changes that have taken place in the past few decades regarding literature in the design, analysis and application of various numerical algorithms for solving hyperbolic equations. This volume provides concise summaries from experts in different types of algorithms, so that readers can find a variety of algorithms under different situations and readily understand their relative advantages and limitations. Provides detailed, cutting-edge background explanations of existing algorithms and their analysis Ideal for readers working on the theoretical aspects of algorithm development and its numerical analysis Presents a method of different algorithms for specific applications and the relative advantages and limitations of different algorithms for engineers or readers involved in applications Written by leading subject experts in each field who provide breadth and depth of content coverage

Praise for the First Edition ". . . outstandingly appealing with regard to its style, contents, considerations of requirements of practice, choice of examples, and exercises." —Zentrablatt Math ". . . carefully structured with many detailed worked examples . . ." —The Mathematical Gazette ". . . an up-to-date and user-friendly account . . ." —Mathematika An Introduction to Numerical Methods and Analysis addresses the mathematics underlying approximation and scientific computing and successfully explains where approximation methods come from, why they sometimes work (or don't work), and when to use one of the many techniques that are available. Written in a style that emphasizes readability and usefulness for the numerical methods novice, the book begins with basic, elementary material and gradually builds up to more advanced topics. A selection of concepts required for the study of computational mathematics is introduced, and simple approximations using Taylor's Theorem are also treated in some depth. The text includes exercises that run the gamut from simple hand computations, to challenging derivations and minor proofs, to programming exercises. A greater emphasis on applied exercises as well as the cause and effect associated with numerical mathematics is featured throughout the book. An Introduction to Numerical Methods and Analysis is the ideal text for students in advanced undergraduate mathematics and engineering courses who are interested in gaining an understanding of numerical methods and numerical analysis.

Along with finite differences and finite elements, spectral methods are one of the three main methodologies for solving partial differential equations on computers. This book provides a detailed presentation of basic spectral algorithms, as well as a systematical presentation of basic convergence theory and error analysis for spectral methods. Readers of this book will be exposed to a unified framework for designing and analyzing spectral algorithms for a variety of problems, including in particular high-order differential equations and problems in unbounded domains. The book contains a large number of figures which are designed to illustrate various concepts stressed in the book. A set of basic matlab codes has been made available online to help the readers to develop their own spectral codes for their specific applications.

This open access book features a selection of high-quality papers from the presentations at the International Conference on Spectral and High-Order Methods 2018, offering an overview of the depth and breadth of the activities within this important research area. The carefully reviewed papers provide a snapshot of the state of the art, while the extensive bibliography helps initiate new research directions.

This book presents the basic algorithms, the main theoretical results, and some applications of spectral methods. Particular attention is paid to the applications of spectral methods to nonlinear problems arising in fluid dynamics, quantum mechanics, weather prediction, heat conduction and other fields.The book consists of three parts. The first part deals with orthogonal approximations in Sobolev spaces and the stability and convergence of approximations for nonlinear problems, as the mathematical foundation of spectral methods. In the second part, various spectral methods are described, with some applications. It includes Fourier spectral method, Legendre spectral method, Chebyshev spectral method, spectral penalty method, spectral vanishing viscosity method, spectral approximation of isolated solutions, multi-dimensional spectral method, spectral method for high-order equations, spectral-domain decomposition method and spectral multigrid method. The third part is devoted to some recent developments of spectral methods, such as mixed spectral methods, combined spectral methods and spectral methods on the surface.

lead the reader to a theoretical understanding of the subject without neglecting its practical aspects. The outcome is a textbook that is mathematically honest and rigorous and provides its target audience with a wide range of skills in both ordinary and partial differential equations." --Book Jacket.

Completely revised text applies spectral methods to boundary value, eigenvalue, and time-dependent problems, but also covers cardinal functions, matrix-solving methods, coordinate transformations, much more. Includes 7 appendices and over 160 text figures.

The@ first graduate-level textbook to focus on fundamental aspects of numerical methods for stochastic computations, this book describes the class of numerical methods based on generalized polynomial chaos (gPC). These fast, efficient, and accurate methods are an extension of the classical spectral methods of high-dimensional random spaces. Designed to simulate complex systems subject to random inputs, these methods are widely used in many areas of computer science and engineering. The book introduces polynomial approximation theory and probability theory; describes the basic theory of gPC methods through numerical examples and rigorous development; details the procedure for converting stochastic equations into deterministic ones; using both the Galerkin and collocation approaches; and discusses the distinct differences and challenges arising from high-dimensional problems. The last section is devoted to the application of gPC methods to critical areas such as inverse problems and data assimilation. Ideal for use by graduate students and researchers both in the classroom and for self-study, Numerical Methods for Stochastic Computations provides the required tools for in-depth research related to stochastic computations. The first graduate-level textbook to focus on the fundamentals of numerical methods for stochastic computations Ideal introduction for graduate courses or self-study Fast, efficient, and accurate numerical methods Polynomial approximation theory and

probability theory included Basic gPC methods illustrated through examples
This collection of essays explores the ancient affinity between the mathematical and the aesthetic, focusing on fundamental connections between these two modes of reasoning and communicating. From historical, philosophical and psychological perspectives, with particular attention to certain mathematical areas such as geometry and analysis, the authors examine ways in which the aesthetic is ever-present in mathematical thinking and contributes to the growth and value of mathematical knowledge.
Spectral Methods Using Multivariate Polynomials on the Unit Ball is a research level text on a numerical method for the solution of partial differential equations. The authors introduce, illustrate with examples, and analyze 'spectral methods' that are based on multivariate polynomial approximations. The method presented is an alternative to finite element and difference methods for regions that are diffeomorphic to the unit disk, in two dimensions, and the unit ball, in three dimensions. The speed of convergence of spectral methods is usually much higher than that of finite element or finite difference methods. Features Introduces the use of multivariate polynomials for the construction and analysis of spectral methods for linear and nonlinear boundary value problems Suitable for researchers and students in numerical analysis of PDEs, along with anyone interested in applying this method to a particular physical problem One of the few texts to address this area using multivariate orthogonal polynomials, rather than tensor products of univariate polynomials.
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This book focuses on the constructive and practical aspects of spectral methods. It rigorously examines the most important qualities as well as drawbacks of spectral methods in the context of numerical methods devoted to solve non-standard eigenvalue problems. In addition, the book also considers some nonlinear singularly perturbed boundary value problems along with eigenproblems obtained by their linearization around constant solutions. The book is mathematical, poising problems in their proper function spaces, but its emphasis is on algorithms and practical difficulties. The range of applications is quite large. High order eigenvalue problems are frequently beset with numerical ill conditioning problems. The book describes a wide variety of successful modifications to standard algorithms that greatly mitigate these problems. In addition, the book makes heavy use of the concept of pseudospectrum, which is highly relevant to understanding when disaster is imminent in solving eigenvalue problems. It also envisions two classes of applications, the stability of some elastic structures and the hydrodynamic stability of some parallel shear flows. This book is an ideal reference text for professionals (researchers) in applied mathematics, computational physics and engineering. It will be very useful to numerically sophisticated engineers, physicists and chemists. The book can also be used as a textbook in

review courses such as numerical analysis, computational methods in various engineering branches or physics and computational methods in analysis.

This well-written book explains the theory of spectral methods and their application to the computation of viscous incompressible fluid flow, in clear and elementary terms. With many examples throughout, the work will be useful to those teaching at the graduate level, as well as to researchers working in the area.

This textbook is intended to introduce advanced undergraduate and early-career graduate students to the field of numerical analysis. This field pertains to the design, analysis, and implementation of algorithms for the approximate solution of mathematical problems that arise in applications spanning science and engineering, and are not practical to solve using analytical techniques such as those taught in courses in calculus, linear algebra or differential equations.Topics covered include computer arithmetic, error analysis, solution of systems of linear equations, least squares problems, eigenvalue problems, nonlinear equations, optimization, polynomial interpolation and approximation, numerical differentiation and integration, ordinary differential equations, and partial differential equations. For each problem considered, the presentation includes the derivation of solution techniques, analysis of their efficiency, accuracy and robustness, and details of their implementation, illustrated through the Python programming language.This text is suitable for a year-long sequence in numerical analysis, and can also be used for a one-semester course in numerical linear algebra.

The primary objective of the course presented here is orientation for those interested in applying mathematics, but the course should also be of value or in using math to those interested in mathematical research and teaching ematics in some other professional context. The course should be suitable for college seniors and graduate students, as well as for college juniors who have had mathematics beyond the basic calculus sequence. Maturity is more significant than any formal prerequisite. The presentation involves a number of topics that are significant for applied mathematics but that normally do not appear in the curriculum or are depicted from an entirely different point of view. These topics include engineering simulations, the experience patterns of the exact sciences, the conceptual nature of pure mathematics and its relation to applied mathe matics, the historical development of mathematics, the associated conceptual aspects of the exact sciences, and the metaphysical implications of mathe matical scientific theories. We will associate topics in mathematics with areas of application. This presentation corresponds to a certain logical structure. But there is an enormous wealth of intellectual development available, and this permits considerable flexibility for the instructor in curricula and emphasis. The prime objective is to encourage the student to contact and utilize this rich heritage. Thus, the student's activity is critical, and it is also critical that this activity be precisely formulated and communicated.

A balanced guide to the essential techniques for solving elliptic partial differential equations Numerical Analysis of Partial Differential Equations provides a comprehensive, self-contained treatment of the quantitative methods used to solve elliptic partial differential equations (PDEs), with a focus on the efficiency as well as the error of the presented methods. The author utilizes coverage of theoretical PDEs, along with the nu merical solution of linear systems and various examples and exercises, to supply readers with an introduction to the essential concepts in the numerical analysis of PDEs. The book presents the three main discretization methods of elliptic PDEs: finite difference, finite elements, and spectral methods. Each topic has its own devoted chapters and is discussed alongside additional key topics, including: The mathematical theory of elliptic PDEs Numerical linear algebra Time-dependent PDEs Multigrid and domain decomposition PDEs posed on infinite domains The book concludes with a discussion of the methods for nonlinear problems, such as Newton's method, and addresses the importance of hands-on work to facilitate learning. Each chapter concludes with a set of exercises, including theoretical and programming problems, that allows readers to test their understanding of the presented theories and techniques. In addition, the book discusses important nonlinear problems in many fields of science and engineering, providing information as to how they can serve as computing projects across various disciplines. Requiring only a preliminary understanding of analysis, Numerical Analysis of Partial Differential Equations is suitable for courses on numerical PDEs at the upper-undergraduate and graduate levels. The book is also appropriate for students majoring in the mathematical sciences and engineering.

This multi-volume handbook is the most up-to-date and comprehensive reference work in the field of fractional calculus and its numerous applications. This third volume collects authoritative chapters covering several numerical aspects of fractional calculus, including time and space fractional derivatives, finite differences and finite elements, and spectral, meshless, and particle methods.

This paper reviews some operators that are used in the numerical analysis of spectral and spectral element methods. We motivate the introduction of these different operators and sketch their approximation properties. Finally we apply them to derive optimal error estimates for spectral type approximations of the solution of elliptic partial differential equations.

Revision of: Spectral/hp element methods for CFD. 1999.

Spectral methods refer to the use of eigenvalues, eigenvectors, singular values and singular vectors. They are widely used in Engineering, Applied Mathematics and Statistics. More recently, spectral methods have found numerous applications in Computer Science to "discrete" as well "continuous" problems. Spectral Algorithms describes modern applications of spectral methods, and novel algorithms for estimating spectral parameters. The first part of the book presents applications of spectral methods to problems from a variety of topics including combinatorial optimization, learning and clustering. The second part of the book is motivated by efficiency considerations. A feature of many modern applications is the massive amount of input data. While sophisticated algorithms for matrix computations have been developed over a century, a more recent development is algorithms based on "sampling on the y" from massive matrices. Good estimates of singular values and low rank approximations of the whole matrix can be provably derived from a sample. The main emphasis in the second part of the book is to present these sampling methods with rigorous error bounds. It also presents recent extensions of spectral methods from matrices to tensors and their applications to some combinatorial optimization problems.

/homepage/sac/cam/na2000/index.html7-Volume Set now available at special set price ! Over the second half of the 20th century the subject area loosely referred to as numerical analysis of partial differential equations (PDEs) has undergone unprecedented development. At its practical end, the vigorous growth and steady diversification of the field were stimulated by the demand for accurate and reliable tools for computational modelling in physical sciences and engineering, and by the rapid development of computer hardware and architecture. At the more theoretical end, the analytical insight into the underlying stability and accuracy properties of computational algorithms for PDEs was deepened by building upon recent progress in mathematical analysis and in the theory of PDEs. To embark on a comprehensive review of the field of numerical analysis of partial differential equations within a single volume of this journal would have been an impossible task. Indeed, the 16 contributions included here, by some of the foremost world authorities in the subject, represent only a small sample of the major developments. We hope that these articles will, nevertheless, provide the reader with a stimulating glimpse into this diverse, exciting and important field. The opening paper by Thomée reviews the history of numerical analysis of PDEs, starting with the 1928 paper by Courant, Friedrichs and Lewy on the solution of problems of mathematical physics by means of finite differences. This excellent survey takes the reader through the development of finite differences for elliptic problems from the 1930s, and the intense study of finite differences for general initial value problems during the 1950s and 1960s. The formulation of the concept of stability is explored in the Lax equivalence theorem and the Kreiss matrix lemmas. Reference is made to the introduction of the finite element method by structural engineers, and a description is given of the subsequent development and mathematical analysis of the finite element method with piecewise polynomial approximating functions. The penultimate section of Thomée's survey deals with `other classes of approximation methods', and this covers methods such as collocation methods, spectral methods, finite volume methods and boundary integral methods. The final section is devoted to numerical linear algebra for elliptic problems. The next three papers, by Bialecki and Fairweather, Hesthaven and Gottlieb and Dahmen, describe, respectively, spline collocation methods, spectral methods and wavelet methods. The work by Bialecki and Fairweather is a comprehensive overview of orthogonal spline collocation from its first appearance to the latest mathematical developments and applications. The emphasis throughout is on problems in two space dimensions. The paper by Hesthaven and Gottlieb presents a review of Fourier and Chebyshev pseudospectral methods for the solution of hyperbolic PDEs. Particular emphasis is placed on the treatment of boundaries, stability of time discretisations, treatment of non-smooth solutions and multidomain techniques. The paper gives a clear view of the advances that have been made over the last decade in solving hyperbolic problems by means of spectral methods, but it shows that many critical issues remain open. The paper by Dahmen reviews the recent rapid growth in the use of wavelet methods for PDEs. The author focuses on the use of adaptivity, where significant successes have recently been achieved. He describes the potential weaknesses of wavelet methods as well as the perceived strengths, thus giving a balanced view that should encourage the study of wavelet methods.

Mathematics of Computing -- Numerical Analysis.

Following up the seminal Spectral Methods in Fluid Dynamics, Spectral Methods: Evolution to Complex Geometries and Applications to Fluid Dynamics contains an extensive survey of the essential algorithmic and theoretical aspects of spectral methods for complex geometries. These types of spectral methods were only just emerging at the time the earlier book was published. The discussion of spectral algorithms for linear and nonlinear fluid dynamics stability analyses is greatly expanded. The chapter on spectral algorithms for incompressible flow focuses on algorithms that have proven most useful in practice, has much greater coverage of algorithms for two or more non-periodic directions, and shows how to treat outflow boundaries. Material on spectral methods for compressible flow emphasizes boundary conditions for hyperbolic systems, algorithms for simulation of homogeneous turbulence, and improved methods for shock fitting. This book is a companion to Spectral Methods: Fundamentals in Single Domains.

This is a book about spectral methods for partial differential equations: when to use them, how to implement them, and what can be learned from their of spectral methods has evolved rigorous theory. The computational side vigorously since the early 1970s, especially in computationally intensive of the more spectacular applications are applications in fluid dynamics. Some of the power of these discussed here, first in general terms as examples of the methods have been methods and later in great detail after the specifics covered. This book pays special attention to those algorithmic details which are essential to successful implementation of spectral methods. The focus is on algorithms for fluid dynamical problems in transition, turbulence, and aero dynamics. This book does not address specific applications in meteorology, partly because of the lack of experience of the authors in this field and partly because of the coverage provided by Haltiner and Williams (1980). The success of spectral methods in practical computations has led to an increasing interest in their theoretical aspects, especially since the mid-1970s. Although the theory does not yet cover the complete spectrum of applications, the analytical techniques which have been developed in recent years have facilitated the examination of an increasing number of problems of practical interest. In this book we present a unified theory of the mathematical analysis of spectral methods and apply it to many of the algorithms in current use.

This monograph gives a mathematical analysis of spectral methods for mixed initial-boundary value problems. Spectral methods have become increasingly popular in recent years, especially since the development of fast transform methods, with applications in numerical weather prediction, numerical simulations of turbulent flows, and other problems where high accuracy is desired for complicated solutions. The development of a mathematical theory is given that explains why spectral methods work and how well they work.