Logical Foundations Of Mathematics And Computational Complexity A Gentle Introduction Springer Monographs In Mathematics

Since their inception, the Perspectives in Logic and Lecture Notes in Logic series have published seminal works by leading logicians. Many of the original books in the series have been unavailable for years, but they are now in print once again. Logicism, as put forward by Bertrand Russell, was predicated on a belief that all of mathematics can be deduced from a very small number of fundamental logical principles. In this volume, the twenty-third publication in the Lecture Notes in Logic series, Paul C. Gilmore revisits logicism in light of recent advances in mathematical logic and theoretical computer science. Gilmore addresses the need for languages which can be understood by both humans and computers and, using Intensional Type Theory (ITT), provides a unified basis for mathematics and computer science. This yields much simpler foundations for recursion theory and the semantics of computer programs than those currently provided by category theory.

This modern introduction to the foundations of logic and mathematics not only takes theory into account, but also treats in some detail applications that have a substantial impact on everyday life (loans and mortgages, bar codes, public-key cryptography). A first college-level introduction to logic, proofs, sets, number theory, and graph theory, and an excellent self-study reference and resource for instructors.

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This book is the product of a yearlong collaboration at the Institute for Advanced Study. It describes (the beta version of) a new language for mathematics, which may some day replace set theory.

Anyone involved in the philosophy of science is naturally drawn into the study of the foundations of probability. Different interpretations of probability, based on competing philosophical ideas, lead to different statistical techniques, and frequently to mutually contradictory consequences. This unique book presents a new interpretation of probability, rooted in the traditional interpretation that was current in the 17th and 18th centuries. Mathematical models are constructed based on this interpretation, and statistical inference and decision theory are applied, including some examples in artificial intelligence, solving the main foundational problems. Nonstandard analysis is extensively developed for the construction of the models and in some of the proofs. Many nonstandard theorems are proved, some of them new, in particular, a representation theorem that asserts that any stochastic process can be approximated by a process defined over a space with equiprobable outcomes.

This volume honours the life and work of Solomon Feferman, one of the most prominent mathematical logicians of the latter half of the 20th century. In the collection of essays presented here, researchers examine Feferman’s work on mathematical as well as specific methodological and philosophical issues that tie into mathematics. Feferman’s work was largely based in mathematical logic (namely model theory, set theory, proof theory and computability theory), but also branched out into methodological and philosophical issues, making it well known beyond the borders of the mathematics community. With regard to methodological issues, Feferman supported concrete projects. On the one hand, these projects calibrate the proof theoretic strength of subsystems of analysis and set theory and provide ways of overcoming the limitations imposed by Gödel’s incompleteness theorems through appropriate conceptual expansions. On the other, they seek to identify novel axiomatic foundations for mathematical practice, truth theories, and category theory. In his philosophical research, Feferman explored questions such as “What is logic?” and proposed particular positions regarding the foundations of mathematics including, for example, his “conceptual structuralism.” The contributing authors of the volume examine all of the above issues. Their papers are accompanied by an autobiography presented by Feferman that reflects on the evolution and intellectual contexts of his work. The contributing authors critically examine Feferman’s work and, in part, actively expand on his concrete mathematical projects. The volume illuminates Feferman’s distinctive work and, in the process, provides an enlightening perspective on the foundations of mathematics and logic.

The theme of this book is formed by a pair of concepts: the concept of formal language as carrier of the precise expression of meaning, facts and problems, and the concept of algorithm or calculus, i.e. a formally operating procedure for the solution of precisely described questions and problems. The book is a unified introduction to the modern theory of these concepts, to the way in which they developed first in mathematical logic and computability theory and later in automata theory, and to the theory of formal languages and complexity theory. Apart from considering the fundamental themes and classical aspects of these areas, the subject matter has been selected to give priority throughout to the new aspects of traditional questions, results and methods which have developed from the needs or knowledge of computer science and particularly of complexity theory. It is both a textbook for introductory courses in the above-mentioned disciplines as well as a monograph in which further results of new research are systematically presented and where an attempt is made to make explicit the connections and analogies between a variety of concepts and constructions.

Fascinating study of the origin and nature of mathematical thought, including relation of mathematics and science, 20th-century developments, impact of computers, and more.Includes 34 illustrations. 1968 edition."

This SpringerBrief provides an interdisciplinary synthesis based on psychology, logic, mathematics, cognitive science, and the history of science. It presents psychology as a science that suffers from a reduced understanding of the most fundamental logic in our practical-bodily encounters with the world, including with our fellow human beings. The Brief offers a new “dual” logic that is based on the duality between identification and description of objects, including persons. The Brief ties in modern mathematics as a tool that can be used to catch this duality in a precise manner. Featured topics in this Brief include: The emergence of Mechanism. The duality in animal and human subject-object relations. Psychology’s compatibility with natural sciences. Four cornerstones of modern mathematics. The Extensional Method. A New Logical Foundation for Psychology will be of interest to psychologist, philosophers, and mathematicians concerned with basic theoretical

and methodological problems.

This book addresses the logical aspects of the foundations of scientific theories. Even though the relevance of formal methods in the study of scientific theories is now widely recognized and regaining prominence, the issues covered here are still not generally discussed in philosophy of science. The authors focus mainly on the role played by the underlying formal apparatuses employed in the construction of the models of scientific theories, relating the discussion with the so-called semantic approach to scientific theories. The book describes the role played by this metamathematical framework in three main aspects: considerations of formal languages employed to axiomatize scientific theories, the role of the axiomatic method itself, and the way set-theoretical structures, which play the role of the models of theories, are developed. The authors also discuss the differences and philosophical relevance of the two basic ways of aximoatizing a scientific theory, namely Patrick Suppes' set theoretical predicates and the "da Costa and Chuaqui" approach. This book engages with important discussions of the nature of scientific theories and will be a useful resource for researchers and upper-level students working in philosophy of science.

The Logical Foundations of Mathematics offers a study of the foundations of mathematics, stressing comparisons between and critical analyses of the major non-constructive foundational systems. The position of constructivism within the spectrum of foundational philosophies is discussed, along with the exact relationship between topos theory and set theory. Comprised of eight chapters, this book begins with an introduction to first-order logic. In particular, two complete systems of axioms and rules for the first-order predicate calculus are given, one for efficiency in proving metatheorems, and the other, in a "natural deduction" style, for presenting detailed formal proofs. A somewhat novel feature of this framework is a full semantic and syntactic treatment of variable-binding term operators as primitive symbols of logic. Subsequent chapters focus on the origin of modern foundational studies; Gottlob Frege's formal system intended to serve as a foundation for mathematics and its paradoxes; the theory of types; and the Zermelo-Fraenkel set theory. David Hilbert's program and Kurt Gödel's incompleteness theorems are also examined, along with the foundational systems of W. V. Quine and the relevance of categorical algebra for foundations. This monograph will be of interest to students, teachers, practitioners, and researchers in mathematics.

This book treats bounded arithmetic and propositional proof complexity from the point of view of computational complexity. The first seven chapters include the necessary logical background for the material and are suitable for a graduate course. Associated with each of many complexity classes are both a two-sorted predicate calculus theory, with induction restricted to concepts in the class, and a propositional proof system. The complexity classes range from AC0 for the weakest theory up to the polynomial hierarchy. Each bounded theorem in a theory translates into a family of (quantified) propositional tautologies with polynomial size proofs in the corresponding proof system. The theory proves the soundness of the associated proof system. The result is a uniform treatment of many systems in the literature, including Buss's theories for the polynomial hierarchy and many disparate systems for complexity classes such as AC0, AC0(m), TC0, NC1, L, NL, NC, and P.

Since their inception, the Perspectives in Logic and Lecture Notes in Logic series have published seminal works by leading logicians. Many of the original books in the series have been unavailable for years, but they are now in print once again. This volume, the sixth publication in the Lecture Notes in Logic series, collects the proceedings of the conference 'Logical Foundations of Mathematics, Computer Science, and Physics - Kurt Gödel's Legacy', held in Brno, Czech Republic, on the 90th anniversary of Gödel's birth. The broad range of speakers who participated in this event affirms the continuing importance of Gödel's work in logic, physics, and the philosophy and foundations of mathematics and computer science. The papers in this volume range over all these topics and contribute to our present understanding of them.

This book constitutes the refereed proceedings of the International Symposium on Logical Foundations of Computer Science, LFCS 2013, held in San Diego, CA, USA in January 2013. The volume presents 29 revised refereed papers carefully selected by the program committee. The scope of the Symposium is broad and includes constructive mathematics and type theory; logic, automata and automatic structures; computability and randomness; logical foundations of programming; logical aspects of computational complexity; logic programming and constraints; automated deduction and interactive theorem proving; logical methods in protocol and program verification; logical methods in program specification and extraction; domain theory logic; logical foundations of database theory; equational logic and term rewriting; lambda and combinatory calculi; categorical logic and topological semantics; linear logic; epistemic and temporal logics; intelligent and multiple agent system logics; logics of proof and justification; nonmonotonic reasoning; logic in game theory and social software; logic of hybrid systems; distributed system logics; mathematical fuzzy logic; system design logics; and other logics in computer science.

This volume commemorates the life, work and foundational views of Kurt Gödel (1906–78), most famous for his hallmark works on the completeness of first-order logic, the incompleteness of number theory, and the consistency - with the other widely accepted axioms of set theory - of the axiom of choice and of the generalized continuum hypothesis. It explores current research, advances and ideas for future directions not only in the foundations of mathematics and logic, but also in the fields of computer science, artificial intelligence, physics, cosmology, philosophy, theology and the history of science. The discussion is supplemented by personal reflections from several scholars who knew Gödel personally, providing some interesting insights into his life. By putting his ideas and life's work into the context of current thinking and perceptions, this book will extend the impact of Gödel's fundamental work in mathematics, logic, philosophy and other disciplines for future generations of researchers.

The Symposium on Logical Foundations of Computer Science series provides a forum for the fast-growing body of work in the logical foundations of computer science, e.g., those areas of fundamental theoretical logic related to computer science. The LFCS series began with “Logic at Botik,” Pereslavl-Zalessky,1989, which was co-organized by Albert R. Meyer (MIT) and Michael Taitslin (Tver). After that, organization passed to Anil Nerode. Currently LFCS is governed by a Steering Committee consisting of Anil Nerode (General Chair), Stephen Cook, Dirk van Dalen, Yuri Matiyasevich, John McCarthy, J. Alan Robinson, Gerald Sacks, and Dana Scott. The 2009 Symposium on Logical Foundations of Computer Science (LFCS 2009) took place in Howard Johnson Plaza Resort, Deer?eld Beach, Florida, USA, during January 3–6. This volume contains the extended abstracts of talks selected by the Program Committee for presentation at LFCS 2009. The scope of the symposium is broad and contains constructive mathematics and type theory; automata and automatic structures; computability and r- domness; logical foundations of programming; logical aspects of computational complexity; logic programmingand constraints;automated deduction and int- active theorem proving; logical methods in protocol and program veri?cation; logical methods in program speci?cation and extraction; domain theory l- ics; logical foundations of database theory; equational logic and term rewriting; lambda andcombinatorycalculi;categoricallogicandtopologicalsemantics;l- ear logic; epistemic and temporal logics; intelligent and multiple agent system logics; logics of proof and justi?cation; nonmonotonic reasoning; logic in game theory and social software; logic of hybrid systems; distributed system logics;

This book constitutes the refereed proceedings of the International Symposium on Logical Foundations of Computer Science, LFCS 2016, held in Deerfield Beach, FL, USA in January 2016. The 27 revised full papers were carefully reviewed and selected from 46 submissions. The scope of the Symposium is broad and includes constructive mathematics and type theory; homotopy type theory; logic, automata, and automatic structures; computability and randomness; logical foundations of programming; logical aspects of computational complexity; parameterized complexity; logic programming and constraints; automated deduction and interactive theorem proving; logical methods in protocol and program verification; logical methods in program specification and extraction; domain theory logics; logical foundations of database theory; equational logic and term rewriting; lambda and combinatory calculi; categorical logic and topological semantics; linear logic; epistemic and temporal logics; intelligent and multiple-agent system logics; logics of proof and justification; non-monotonic reasoning; logic in game theory and social software; logic of hybrid systems; distributed system logics; mathematical fuzzy logic; system design logics; and other logics in computer science.

First published in 2000. Routledge is an imprint of Taylor & Francis, an informa company.

Mathematical logic is a branch of mathematics that takes axiom systems and mathematical proofs as its objects of study. This book shows how it can also provide a foundation for the development of information science and technology. The first five chapters systematically present the core topics of classical mathematical logic, including the syntax and models of first-order languages, formal inference systems, computability and representability, and Gödel’s theorems. The last five chapters present extensions and developments of classical mathematical logic, particularly the concepts of version sequences of formal theories and their limits, the system of revision calculus, proschemes (formal descriptions of proof methods and strategies) and their properties, and the theory of inductive inference. All of these themes contribute to a formal theory of axiomatization and its application to the process of developing information technology and scientific theories. The book also describes the paradigm of three kinds of language environments for theories and it presents the basic properties required of a meta-language environment. Finally, the book brings these themes together by describing a workflow for scientific research in the information era in which formal methods, interactive software and human invention are all used to their advantage. The second edition of the book includes major revisions on the proof of the completeness theorem of the Gentzen system and new contents on the logic of scientific discovery, R-calculus without cut, and the operational semantics of program debugging. This book represents a valuable reference for graduate and undergraduate students and researchers in mathematics, information science and technology, and other relevant areas of natural sciences. Its first five chapters serve as an undergraduate text in mathematical logic and the last five chapters are addressed to graduate students in relevant disciplines.

Philosophical Approaches to the Foundations of Logic and Mathematics consists of eleven articles addressing various aspects of the "roots" of logic and mathematics, their basic concepts and the mechanisms that work in the practice of their use.

The scope and method of logic as we know it today eminently reflect the ground-breaking developments of set theory and the logical foundations of mathematics at the turn of the 20th century. Unfortunately, little effort has been made to understand the idiosyncrasies of the philosophical context that led to these tremendous innovations in the 19thcentury beyond what is found in the works of mathematicians such as Frege, Hilbert, and Russell. This constitutes a monumental gap in our understanding of the central influences that shaped 19th-century thought, from Kant to Russell, and that helped to create the conditions in which analytic philosophy could emerge. The aim of Logic from Kant to Russell is to document the development of logic in the works of 19th-century philosophers. It contains thirteen original essays written by authors from a broad range of backgrounds—intellectual historians, historians of idealism, philosophers of science, and historians of logic and analytic philosophy. These essays question the standard narratives of analytic philosophy’s past and address concerns that are relevant to the contemporary philosophical study of language, mind, and cognition. The book covers a broad range of influential thinkers in 19th-century philosophy and analytic philosophy, including Kant, Bolzano, Hegel, Herbart, Lotze, the British Algebraists and Idealists, Moore, Russell, the Neo-Kantians, and Frege.

The two main themes of this book, logic and complexity, are both essential for understanding the main problems about the foundations of mathematics. Logical Foundations of Mathematics and Computational Complexity covers a broad spectrum of results in logic and set theory that are relevant to the foundations, as well as the results in computational complexity and the interdisciplinary area of proof complexity. The author presents his ideas on how these areas are connected, what are the most fundamental problems and how they should be approached. In particular, he argues that complexity is as important for foundations as are the more traditional concepts of computability and provability. Emphasis is on explaining the essence of concepts and the ideas of proofs, rather than presenting precise formal statements and full proofs. Each section starts with concepts and results easily explained, and gradually proceeds to more difficult ones. The notes after each section present some formal definitions, theorems and proofs. Logical Foundations of Mathematics and Computational Complexity is aimed at graduate students of all fields of mathematics who are interested in logic, complexity and foundations. It will also be of interest for both physicists and philosophers who are curious to learn the basics of logic and complexity theory.

This volume contains the proceedings of the conference Logical Foundations of Mathematics, Computer Science, and Physics-Kurt Gödel's Legacy, held in Brno, Czech Republic on the 90th anniversary of his birth. The wide and continuing importance of Gödel s work in the logical foundations of mathematics, computer science, and physics is confirmed by the broad range of speakers who participated in making this gathering a scientific event.

The book presents logical foundations for rule-based systems. An attempt has been made to provide an in-depth discussion of logical and other aspects of such systems, including languages for knowledge representation, inference mechanisms, inference control, design and verification. The ultimate goal was to provide a deeper theoretical insight into the nature of rule-based systems and put together the most complete presentation including details so frequently skipped in typical textbooks. The book may be useful to potentially wide audience, but it is aimed at providing specific knowledge for graduate, post-graduate and Ph.D. students, as well as knowledge engineers and research workers involved in the domain of AI. It also constitutes a summary of the Author’s research and experience gathered through several years of his research work.

Treats politics, economics, technology, and geography as fundamental factors in generating an audience for logic.

Collection of works by Frank Plumpton Ramsey (1903-1930), who made seminal contributions to philosophy, mathematics and economics. Whilst he was acknowledged as a genius by his contemporaries, some of his most important ideas were not appreciated until decades later; now better appreciated, they continue to bear an influence upon contemporary philosophy. His historic significance was to usher in a new phase of analytic philosophy, which initially built upon the logical atomist doctrines of Bertrand Russell and Ludwig Wittgenstein, raising their ideas to a new level of sophistication, but ultimately he became their successor rather than remain a mere acolyte.

This text for the first or second year undergraduate in mathematics, logic, computer science, or social sciences, introduces the reader to logic, proofs, sets, and number theory. It also serves as an excellent independent study reference and resource for instructors. Adapted from Foundations of Logic and Mathematics: Applications to Science and Cryptography © 2002 Birkh?user, this second edition provides a modern introduction to the foundations of logic, mathematics, and computers science, developing the theory that demonstrates construction of all mathematics and theoretical computer science from logic and set theory. The focuses is on foundations, with specific statements of all the associated axioms and rules of logic and set theory, and provides complete details and derivations of formal proofs. Copious references to literature that document historical development is also provided. Answers are found to many questions that usually remain unanswered: Why is the truth table for logical implication so unintuitive? Why are there no recipes to design proofs? Where do these numerous mathematical rules come from? What issues in logic, mathematics, and computer science still remain unresolved? And the perennial question: In what ways are we going to use this material? Additionally, the selection of topics presented reflects many major accomplishments from the twentieth century and includes applications in game theory and Nash's equilibrium, Gale and Shapley's match making algorithms, Arrow's Impossibility Theorem in voting, to name a few. From the reviews of the first edition: "...All the results are proved in full detail from first principles...remarkably, the arithmetic laws on the rational numbers are proved, step after step, starting from the very definitions!...This is a valuable reference text and a useful companion for anybody wondering how basic mathematical concepts can be rigorously developed within set theory." —MATHEMATICAL REVIEWS "Rigorous and modern in its theoretical aspect, attractive as a detective novel in its applied aspects, this paper book deserves the attention of both beginners and advanced students in mathematics, logic and computer sciences as well as in social sciences." —Zentralblatt MATH

Mathematical logic grew out of philosophical questions regarding the foundations of mathematics, but logic has now outgrown its philosophical roots, and has become an integral part of mathematics in general. This book is designed for students who plan to specialize in logic, as well as for those who are interested in the applications of logic to other areas of mathematics. Used as a text, it could form the basis of a beginning graduate-level course. There are three main chapters: Set Theory, Model Theory, and Recursion Theory. The Set Theory chapter describes the set-theoretic foundations of all of mathematics, based on the ZFC axioms. It also covers technical results about the Axiom of Choice, well-orderings, and the theory of uncountable cardinals. The Model Theory chapter discusses predicate logic and formal proofs, and covers the Completeness, Compactness, and Lowenheim-Skolem Theorems, elementary submodels, model completeness, and applications to algebra. This chapter also continues the foundational issues begun in the set theory chapter. Mathematics can now be viewed as formal proofs from ZFC. Also, model theory leads to models of set theory. This includes a discussion of absoluteness, and an analysis of models such as H( ) and R( ). The Recursion Theory chapter develops some basic facts about computable functions, and uses them to prove a number of results of foundational importance; in particular, Church's theorem on the undecidability of logical consequence, the incompleteness theorems of Godel, and Tarski's theorem on the non-definability of truth.

Authored by an outstanding collection of leading theorists and researchers from a range of disciplines, this book details the inadequacies of classical logic in its handling of ordinary language and reveals the prospects for applying a synthesis of logic and psychology to cognitive psychology, linguistics, and the philosophy of language.

This book constitutes the refereed proceedings of the International Symposium on Logical Foundations of Computer Science, LFCS 2018, held in Deerfield Beach, FL, USA, in January 2018. The 22 revised full papers were carefully reviewed and selected from 22 submissions. The scope of the Symposium is broad and includes constructive mathematics and type theory; homotopy type theory; logic, automata, and automatic structures; computability and randomness; logical foundations of programming; logical aspects of computational complexity; parameterized complexity; logic programming and constraints; automated deduction and interactive theorem proving; logical methods in protocol and program verification; logical methods in program specification and extraction; domain theory logics; logical foundations of database theory; equational logic and term rewriting; lambda andcombinatory calculi; categorical logic and topological semantics; linear logic; epistemic and temporal logics; intelligent and multiple-agent system logics; logics of proof and justification; non-monotonic reasoning; logic in game theory and social software; logic of hybrid systems; distributed system logics; mathematical fuzzy logic; system design logics; and other logics in computer science.

This book constitutes the refereed proceedings of the International Symposium on Logical Foundations of Computer Science, LFCS 2020, held in Deerfield Beach, FL, USA, in January 2020. The 17 revised full papers were carefully reviewed and selected from 30 submissions. The scope of the Symposium is broad and includes constructive mathematics and type theory; homotopy type theory; logic, automata, and automatic structures; computability and randomness; logical foundations of programming; logical aspects of computational complexity; parameterized complexity; logic programming and constraints; automated deduction and interactive theorem proving; logical methods in protocol and program verification; logical methods in program specification and extraction; domain theory logics; logical foundations of database theory; equational logic and term rewriting; lambda and combinatory calculi; categorical logic and topological semantics; linear logic; epistemic and temporal logics; intelligent and multiple-agent system logics; logics of proof and justification; non-monotonic reasoning; logic in game theory and social software; logic of hybrid systems; distributed system logics; mathematical fuzzy logic; system design logics; other logics in computer science.