Introduction To Hilbert Space And The Theory Of Spectral Multiplicity Ams Chelsea Publishing

Numerous worked examples and exercises highlight this unified treatment. Simple explanations of difficult subjects make it accessible to undergraduates as well as an ideal self-study guide. 1990 edition.

From Euclidian to Hilbert Spaces analyzes the transition from finite dimensional Euclidian spaces to infinite-dimensional Hilbert spaces, a notion that can sometimes be difficult for non-specialists to grasp. The focus is on the parallels and differences between the properties of the finite and infinite dimensions, noting the fundamental importance of coherence between the algebraic and topological structure, which makes Hilbert spaces the infinite-dimensional objects most closely related to Euclidian spaces. The common thread of this book is the Fourier transform, which is examined starting from the discrete Fourier transform (DFT), along with its applications in signal and image processing, passing through the Fourier series and finishing with the use of the Fourier transform to solve differential equations. The geometric structure of Hilbert spaces and the most significant properties of bounded linear operators in these spaces are also covered extensively. The theorems are presented with detailed proofs as well as meticulously explained exercises and solutions, with the aim of illustrating the variety of applications of the theoretical results.

Building on the success of the two previous editions, Introduction to Hilbert Spaces with Applications, Third Edition, offers an overview of the basic ideas and results of Hilbert space theory and functional analysis. It acquaints students with the Lebesgue integral, and includes an enhanced presentation of results and proofs. Students and researchers will benefit from the wealth of revised examples in new, diverse applications as they apply to optimization, variational and control problems, and problems in approximation theory, nonlinear instability, and bifurcation. The text also includes a popular chapter on wavelets that has been completely updated. Students and researchers agree that this is the definitive text on Hilbert Space theory. Updated chapter on wavelets Improved presentation on results and proof Revised examples and updated applications Completely updated list of references

This reference text, now in its second edition, offers a modern unifying presentation of three basic areas of nonlinear analysis: convex analysis, monotone operator theory, and the fixed point theory of nonexpansive operators. Taking a unique comprehensive approach, the theory is developed from the ground up, with the rich connections and interactions between the areas as the central focus, and it is illustrated by a large number of examples. The Hilbert space setting of the material offers a wide range of applications while avoiding the technical difficulties of general Banach spaces. The authors have also drawn upon recent advances and modern tools to simplify the proofs of key results making the book more accessible to a broader range of scholars and users. Combining a strong emphasis on applications with exceptionally lucid writing and an abundance of exercises, this text is of great value to a large audience including pure and applied mathematicians as well as researchers in engineering, data science, machine learning, physics, decision sciences, economics, and inverse problems. The second edition of Convex Analysis and Monotone Operator Theory in Hilbert Spaces greatly expands on the first edition, containing over 140 pages of new material, over 270 new results, and more than 100 new exercises. It features a new chapter on proximity operators including two sections on proximity operators of matrix functions, in addition to several new sections distributed throughout the original chapters. Many existing results have been improved, and the list of references has been updated. Heinz H. Bauschke is a Full Professor of Mathematics at the Kelowna campus of the University of British Columbia, Canada. Patrick L. Combettes, IEEE Fellow, was on the faculty of the City University of New York and of

Université Pierre et Marie Curie – Paris 6 before joining North Carolina State University as a Distinguished Professor of Mathematics in 2016.

From the Preface: "This book was written for the active reader. The first part consists of problems, frequently preceded by definitions and motivation, and sometimes followed by corollaries and historical remarks... The second part, a very short one, consists of hints... The third part, the longest, consists of solutions: proofs, answers, or contructions, depending on the nature of the problem.... This is not an introduction to Hilbert space theory. Some knowledge of that subject is a prerequisite: at the very least, a study of the elements of Hilbert space theory should proceed concurrently with the reading of this book."

This textbook is an introduction to functional analysis suited to final year undergraduates or beginning graduates. Its various applications of Hilbert spaces, including least squares approximation, inverse problems, and Tikhonov regularization, should appeal not only to mathematicians interested in applications, but also to researchers in related fields. Functional Analysis adopts a self-contained approach to Banach spaces and operator theory that covers the main topics, based upon the classical sequence and function spaces and their operators. It assumes only a minimum of knowledge in elementary linear algebra and real analysis; the latter is redone in the light of metric spaces. It contains more than a thousand worked examples and exercises, which make up the main body of the book.

The primarily objective of the book is to serve as a primer on the theory of bounded linear operators on separable Hilbert space. The book presents the spectral theorem as a statement on the existence of a unique continuous and measurable functional calculus. It discusses a proof without digressing into a course on the Gelfand theory of commutative Banach algebras. The book also introduces the reader to the basic facts concerning the various von Neumann–Schatten ideals, the compact operators, the trace-class operators and all bounded operators.

In an elegant and concise fashion, this book presents the concepts of functional analysis required by students of mathematics and physics. It begins with the basics of normed linear spaces and quickly proceeds to concentrate on Hilbert spaces, specifically the spectral theorem for bounded as well as unbounded operators in separable Hilbert spaces. While the first two chapters are devoted to basic propositions concerning normed vector spaces and Hilbert spaces, the third chapter treats advanced topics which are perhaps not standard in a first course on functional analysis. It begins with the Gelfand theory of commutative Banach algebras, and proceeds to the Gelfand-Naimark theorem on commutative C*-algebras. A discussion of representations of C*-algebras follows, and the final section of this chapter is devoted to the Hahn-Hellinger classification of separable representations of commutative C*-algebras. After this detour into operator algebras, the fourth chapter reverts to more standard operator theory in Hilbert space, dwelling on topics such as the spectral theorem for normal operators, the polar decomposition theorem, and the Fredholm theory for compact operators. A brief introduction to the theory of unbounded operators on Hilbert space is given in the fifth and final chapter. There is a voluminous appendix whose purpose is to fill in possible gaps in the reader's background in various areas such as linear algebra, topology, set theory and measure theory. The book is interspersed with many exercises, and hints are provided for the solutions to the more challenging of these.

An accessible introduction to Hilbert spaces, combining the theory with applications of Hilbert methods in signal processing.

This English edition is almost identical to the German original Lineare Operatoren in Hilbertriiumen, published by B. G. Teubner, Stuttgart in 1976. A few proofs have been simplified, some additional exercises have been included, and a small number of new results has been added (e.g., Theorem 11.11 and Theorem 11.23). In addition a great number of minor errors has been corrected. Frankfurt, January 1980 J. Weidmann vii Preface to the German edition The purpose of this book is to give an introduction to the theory of linear operators on Hilbert spaces and then to proceed to the interesting applica tions of differential operators to mathematical physics. Besides the usual introductory courses common to both mathematicians and physicists, only a fundamental knowledge of complex analysis and of ordinary differential equations is assumed. The most important results of Lebesgue integration theory, to the extent that they are used in this book, are compiled with complete proofs in Appendix A. I hope therefore that students from the fourth semester on will be able to read this book without major difficulty. However, it might also be of some interest and use to the teaching and research mathematician or physicist, since among other things it makes easily accessible several new results of the spectral theory of differential operators. This book offers an elementary and engaging introduction to operator theory on the Hardy-Hilbert space. It provides a firm foundation for the study of all spaces of analytic functions and of the operators on them. Blending techniques from "soft" and "hard" analysis, the book contains clear and beautiful proofs. There are numerous exercises at the end of each chapter, along with a brief guide for further study which includes references to applications to topics in engineering.

The book concisely presents the fundamental aspects of the theory of operators on Hilbert spaces. The topics covered include functional calculus and spectral theorems, compact operators, trace class and Hilbert-Schmidt operators, self-adjoint extensions of symmetric operators, and one-parameter groups of operators. The exposition of the material on unbounded operators is based on a novel tool, called the z-transform, which provides a way to encode full information about unbounded operators in bounded ones, hence making many technical aspects of the theory less involved.

Concise introductory treatment consists of three chapters: The Geometry of Hilbert Space, The Algebra of Operators, and The Analysis of Spectral Measures. A background in measure theory is the sole prerequisite. 1957 edition.

This book treats the fundamental mathematical properties that hold for a family of Gaussian random variables.

The notion of a Hilbert space is a central idea in functional analysis and this text demonstrates its applications in numerous branches of pure and applied mathematics. The book presents an introduction to the geometry of Hilbert spaces and operator theory, targeting graduate and senior undergraduate students of mathematics. Major topics discussed in the book are inner product spaces, linear operators, spectral theory and special classes of operators, and Banach spaces. On vector spaces, the structure of inner product is imposed. After discussing geometry of Hilbert spaces, its applications to diverse branches of mathematics have been studied. Along the way are introduced orthogonal polynomials and their use in Fourier series and approximations. Spectrum of an operator is the key to the understanding of the operator. Properties of the spectrum of different classes of operators, such as normal operators, self-adjoint operators, unitaries, isometries and compact operators have been discussed. A large number of examples of operators, along with their spectrum and its splitting into point spectrum, continuous spectrum, residual spectrum, approximate point spectrum and compression spectrum, have been worked out. Spectral theorems for self-adjoint operators, and normal operators, follow the spectral theorem for compact normal operators. The book also discusses invariant subspaces with special attention to the Volterra operator and unbounded operators. In order to make the text as accessible as possible, motivation for the topics is introduced and a greater amount of explanation than is usually found in standard texts on the subject is provided. The abstract theory in the book is supplemented with concrete examples. It is expected that these features will help the reader get a good grasp of the topics discussed. Hints and solutions to all the problems are collected at the end of the book. Additional features are introduced in the book when it becomes imperative. This spirit is kept alive throughout the book.

Completely self-contained ... All proofs are given in full detail ... recommended for unassisted reading by beginners ... For teaching purposes this book is ideal. --Proceedings of the Edinburgh Mathematical Society The book is easy to read and, although the author had in mind graduate students, most of it is obviously appropriate for an advanced undergraduate course. It is also a book which a reasonably good student might read on his own. --Mathematical Reviews This textbook evolved from a set of course notes for first- or second-year graduate students in mathematics and related fields such as physics. It presents, in a self-contained way, various aspects of geometry and analysis of Hilbert spaces, including the spectral theorem for compact operators. Over 400 exercises provide examples and counter-examples for definitions and theorems in the book, as well as generalization of some material in the text. Aside from being an exposition of basic material on Hilbert space, this book may also serve as an introduction to other areas of functional analysis. The only prerequisite for understanding the material is a standard foundation in advanced calculus. The main notions of linear algebra, such as vector spaces, bases, etc., are explained in the first chapter of the book.

This textbook is an introduction to the theory of Hilbert space and its applications. The notion of Hilbert space is central in functional analysis and is used in numerous branches of pure and applied mathematics. Dr Young has stressed applications of the theory, particularly to the solution of partial differential equations in mathematical physics and to the approximation of functions in complex analysis. Some basic familiarity with real analysis, linear algebra and metric spaces is assumed, but otherwise the book is self-contained. It is based on courses given at the University of Glasgow and contains numerous examples and exercises (many with solutions). Thus it will make an excellent first course in Hilbert space theory at either undergraduate or graduate level and will also be of interest to electrical engineers and physicists, particularly those involved in control theory and filter design.

Reproducing kernel Hilbert spaces have developed into an important tool in many areas, especially statistics and machine learning, and they play a valuable role in complex analysis, probability, group representation theory, and the theory of integral operators. This unique text offers a unified overview of the topic, providing detailed examples of applications, as well as covering the fundamental underlying theory, including chapters on interpolation and approximation, Cholesky and Schur operations on kernels, and vector-valued spaces. Self-contained and accessibly written, with exercises at the end of each chapter, this unrivalled treatment of the topic serves as an ideal introduction for graduate students across mathematics, computer science, and engineering, as well as a useful reference for researchers working in functional analysis or its applications.

Physics has long been regarded as a wellspring of mathematical problems. Mathematical Methods in Physics is a self-contained presentation, driven by historic motivations, excellent examples, detailed proofs, and a focus on those parts of mathematics that are needed in more ambitious courses on quantum mechanics and classical and quantum field theory. Aimed primarily at a broad community of graduate students in mathematics, mathematical physics, physics and engineering, as well as researchers in these disciplines.

This book gives a comprehensive introduction to modern quantum mechanics, emphasising the underlying Hilbert space theory and generalised function theory. All the major modern techniques and approaches used in quantum mechanics are introduced, such as Berry phase, coherent and squeezed states, quantum computing, solitons and quantum mechanics. Audience: The book is suitable for graduate students in physics and mathematics.

Accessible text covering core functional analysis topics in Hilbert and Banach spaces, with detailed proofs and 200 fully-worked exercises.

This self-contained work on Hilbert space operators takes a problem-solving approach to the subject, combining theoretical results with a wide variety of exercises that range from the straightforward to the state-of-the-art. Complete solutions to all problems are provided. The text covers the basics of bounded linear operators on a Hilbert space and gradually progresses to more advanced topics in spectral theory and quasireducible operators. Written in a motivating and rigorous style, the work has few prerequisites beyond elementary functional analysis, and will appeal to graduate students and researchers in mathematics, physics, engineering, and related disciplines.

This classic textbook by two mathematicians from the USSR's prestigious Kharkov Mathematics Institute introduces linear operators in Hilbert space, and presents in detail the geometry of Hilbert space and the spectral theory of unitary and self-adjoint operators. It is directed to students at graduate and advanced undergraduate levels, but because of the exceptional clarity of its theoretical presentation and the inclusion of results obtained by Soviet mathematicians, it should prove invaluable for every mathematician and physicist. 1961, 1963 edition.

An Introduction to Hilbert SpaceCambridge University Press

First-ever comprehensive introduction to the major new subject of quantum computing and quantum information.

Historically, nonclassical physics developed in three stages. First came a collection of ad hoc assumptions and then a cookbook of equations known as "quantum mechanics". The equations and their philosophical underpinnings were then collected into a model based on the mathematics of Hilbert space. From the Hilbert space model came the abstaction of "quantum logics". This book explores all three stages, but not in historical order. Instead, in an effort to illustrate how physics and abstract mathematics influence each other we hop back and forth between a purely mathematical development of Hilbert space, and a physically motivated definition of a logic, partially linking the two throughout, and then bringing them together at the deepest level in the last two chapters. This book should be accessible to undergraduate and beginning graduate students in both mathematics and physics. The only strict prerequisites are calculus and linear algebra, but the level of mathematical sophistication assumes at least one or two intermediate courses, for example in mathematical analysis or advanced calculus. No background in physics is assumed.

Continuing on the success of the previous edition, Introduction to Hilbert Spaces with Applications, Second Edition , offers an overview of the basic ideas and results of Hilbert space theory and functional analysis. It acquaints students with the Lebesque integral, and includes an enhanced presentation of results and proofs. Students and researchers benefit from the wealth of revised examples in new, diverse applications as they apply to optimization, variational and control problems, and problems in approximation theory, nonlinear instability, and bifurcation. The text also includes a new, well-researched chapter on wavelets. Students and researchers agree that this is the definitive text on Hilbert Space theory.

This monograph, aimed at graduate students and researchers, explores the use of Hilbert space methods in function theory. Explaining how operator theory interacts with function theory in one and several variables, the authors journey from an accessible explanation of the techniques to their uses in cutting edge research.

This book first rigorously develops the theory of reproducing kernel Hilbert spaces. The authors then discuss the Pick problem of finding the function of smallest H^∞ norm that has specified values at a finite number of points in the disk. Their viewpoint is to consider H^∞ as the multiplier algebra of the Hardy space and to use Hilbert space techniques to solve the problem. This approach generalizes to a wide collection of spaces. The authors then consider the interpolation problem in the space of bounded analytic functions on the bidisk and give a complete description of the solution. They then consider very general interpolation problems. The book includes developments of all the theory that is needed, including operator model theory, the Arveson extension theorem, and the hereditary functional calculus.

North-Holland Series in Applied Mathematics and Mechanics, Volume 6: Introduction to Spectral Theory in Hilbert Space focuses on the mechanics, principles, and approaches involved in spectral theory in Hilbert space. The publication first elaborates on the concept and specific geometry of Hilbert space and bounded linear operators. Discussions focus on projection and adjoint operators, bilinear forms, bounded linear mappings, isomorphisms, orthogonal subspaces, base, subspaces, finite dimensional Euclidean space, and normed linear spaces. The text then takes a look at the general theory of linear operators and spectral analysis of compact linear operators, including spectral decomposition of a compact selfadjoint operator, weakly convergent sequences, spectrum of a compact linear operator, and eigenvalues of a linear operator. The manuscript ponders on the spectral analysis of bounded linear operators and unbounded selfadjoint operators. Topics include spectral decomposition of an unbounded selfadjoint operator and bounded normal operator, functions of a unitary operator, step functions of a bounded selfadjoint operator, polynomials in a bounded operator, and order relation for bounded selfadjoint operators. The publication is a valuable source of data for mathematicians and researchers interested in spectral theory in Hilbert space.

The description for this book, An Introduction to Linear Transformations in Hilbert Space. (AM-4), Volume 4, will be forthcoming.

This book is based on lectures given in a one-quarter course at UCLA. The aim. is to present som.e of the basic concepts and techniques of Functional Analys.is of relevance to optim.ization problem.s in Control. Com.m.unication and other areas in System. Science. The students are expected to have had an introductory course in Hilbert Space theory. Som.e effort has been m.ade to be self-contained m.ainly in order that the vocabularly used can be clarified. A m.inim.al bibliography is appended. The author is indebted to Jiri Ruzicka and Jerom.e Mersky for help with proof-reading. Profes sor L. Berkovitz looked over and m.ade m.any helpful corn.rn.ents on parts of an early version. Thanks are also due to Trudy Cook for typing the m.anuscript. Grateful acknowledgem.ent is also m.ade of partial support under AFOSR Grant No. 68-1408, Applied Mathem.atics Division, United Stat s Air Force.

This book presents basic elements of the theory of Hilbert spaces and operators on Hilbert spaces, culminating in a proof of the spectral theorem for compact, self-adjoint operators on separable Hilbert spaces. It exhibits a construction of the space of pth power Lebesgue integrable functions by a completion procedure with respect to a suitable norm in a space of continuous functions, including proofs of the basic inequalities of Hölder and Minkowski. The Lp-spaces thereby emerges in direct analogy with a construction of the real numbers from the rational numbers. This allows grasping the main ideas more rapidly. Other important Banach spaces arising from function spaces and sequence spaces are also treated. In this second edition, I have expanded the material on normed vector spaces and their operators presented in Chapter 1 to include proofs of the Open Mapping Theorem, the Closed Graph Theorem and the Hahn–Banach Theorem. The material on operators between normed vector spaces is further expanded in a new Chapter 6, which presents the basic elements of the theory of Fredholm operators on general Banach spaces, not only on Hilbert spaces. This requires that we develop the theory of dual operators between Banach spaces to replace the use of adjoint operators between Hilbert spaces. With the addition of the new material on normed vector spaces and their operators, the book can serve as a general introduction to functional analysis viewed as a theory of infinite dimensional linear spaces and linear operators acting on them.

Hilbert space theory is an invaluable mathematical tool in numerous signal processing and systems theory applications. Hilbert spaces satisfying certain additional properties are known as Reproducing Kernel Hilbert Spaces (RKHSs). This primer gives a gentle and novel introduction to RKHS theory. It also presents several classical applications. It concludes by focusing on recent developments in the machine learning literature concerning embeddings of random variables. Parenthetical remarks are used to provide greater technical detail, which some readers may welcome, but they may be ignored without compromising the cohesion of the primer. Proofs are there for those wishing to gain experience at working with RKHSs; simple proofs are preferred to short, clever, but otherwise uninformative proofs. Italicised comments appearing in proofs provide intuition or orientation or both. A Primer on Reproducing Kernel Hilbert Spaces empowers readers to recognize when and how RKHS theory can profit them in their own work.

This book offers an essential introduction to the theory of Hilbert space, a fundamental tool for non-relativistic quantum mechanics. Linear, topological, metric, and normed spaces are all addressed in detail, in a rigorous but reader-friendly fashion. The rationale for providing an introduction to the theory of Hilbert space, rather than a detailed study of Hilbert space theory itself, lies in the strenuous mathematics demands that even the simplest physical cases entail. Graduate courses in physics rarely offer enough time to cover the theory of Hilbert space and operators, as well as distribution theory, with sufficient mathematical rigor. Accordingly, compromises must be found between full rigor and the practical use of the instruments. Based on one of the authors’s lectures on functional analysis for graduate students in physics, the book will equip readers to approach Hilbert space and, subsequently, rigged Hilbert space, with a more practical attitude. It also includes a brief introduction to topological groups, and to other mathematical structures akin to Hilbert space. Exercises and solved problems accompany the main text, offering readers opportunities to deepen their understanding. The topics and their presentation have been chosen with the goal of quickly, yet rigorously and effectively, preparing readers for the intricacies of Hilbert space. Consequently, some topics, e.g., the Lebesgue integral, are treated in a somewhat unorthodox manner. The book is ideally suited for use in upper undergraduate and lower graduate courses, both in Physics and in Mathematics.

The goal of this textbook is to provide an introduction to the methods and language of functional analysis, including Hilbert spaces, Fredholm theory for compact operators, and spectral theory of self-adjoint operators. It also presents the basic theorems and methods of abstract functional analysis and a few applications of these methods to Banach algebras and the theory of unbounded self-adjoint operators. The text corresponds to material for two semester courses (Part I and Part II, respectively), and it is as self-contained as possible. The only prerequisites for the first part are minimal amounts of linear algebra and calculus. However, for the second course (Part II), it is useful to have some knowledge of topology and measure theory. Each chapter is followed by numerous exercises, whose solutions are given at the end of the book.