Geometry Of Quantum Theory

* Invited articles in differential geometry and mathematical physics in honor of Hideki Omori * Focus on recent trends and future directions in symplectic and Poisson geometry, global analysis, Lie group theory, quantizations and noncommutative geometry, as well as applications of PDEs and variational methods to geometry * Will appeal to graduate students in mathematics and quantum mechanics; also a reference

Describes random geometry and applications to strings, quantum gravity, topological field theory and membrane physics.

The geometric formulation of autonomous Hamiltonian mechanics in the terms of symplectic and Poisson manifolds is generally accepted. This book provides the geometric formulation of non-autonomous mechanics in a general setting of time-dependent coordinate and reference frame transformations.

Quantum information theory is a branch of science at the frontier of physics, mathematics, and information science, and offers a variety of solutions that are impossible using classical theory. This book provides a detailed introduction to the key concepts used in processing quantum information and reveals that quantum mechanics is a generalisation of classical probability theory. The second edition contains new sections and entirely new chapters: the hot topic of multipartite entanglement; in-depth discussion of the discrete structures in finite dimensional Hilbert space, including unitary operator bases, mutually unbiased bases, symmetric informationally complete generalized measurements, discrete Wigner function, and unitary designs; the Gleason and Kochen–Specker theorems; the proof of the Lieb conjecture; the measure concentration phenomenon; and the Hastings' non-additivity theorem. This richly-illustrated book will be useful to a broad audience of graduates and researchers interested in quantum information theory. Exercises follow each chapter, with hints and answers supplied.

In the last decade, the development of new ideas in quantum theory, including geometric and deformation quantization, the non-Abelian Berry''s geometric factor, super- and BRST symmetries, non-commutativity, has called into play the geometric techniques based on the deep interplay between algebra, differential geometry and topology. The book aims at being a guide to advanced differential geometric and topological methods in quantum mechanics. Their main peculiarity lies in the fact that geometry in quantum theory speaks mainly the algebraic language of rings, modules, sheaves and categories. Geometry is by no means the primary scope of the book, but it underlies many ideas in modern quantum physics and provides the most advanced schemes of quantization.

In modern mathematical physics, classical together with quantum, geometrical and functional analytic methods are used simultaneously. Non-commutative geometry in particular is becoming a useful tool in quantum field theories. This book, aimed at advanced students and researchers, provides an introduction to these ideas. Researchers will benefit particularly from the extensive survey articles on models relating to quantum gravity, string theory, and non-commutative geometry, as well as Connes' approach to the standard model.

This book continues the fundamental work of Arnold Sommerfeld and David Hestenes formulating theoretical physics in terms of Minkowski space-time geometry. We see how the standard matrix version of the Dirac equation can be reformulated in terms of a real space-time algebra, thus revealing a geometric meaning for the “number i” in quantum mechanics. Next, it is examined in some detail how electroweak theory can be integrated into the Dirac theory and this way interpreted in terms of space-time geometry. Finally, some implications for quantum electrodynamics are considered. The presentation of real quantum electromagnetism is expressed in an addendum. The book covers both the use of the complex and the real languages and allows the reader acquainted with the first language to make a step by step translation to the second one.

Visionary articles explaining approaches to important problems on the interface of pure mathematics and mathematical physics.

The fundamental structure of matter and spacetime at the shortest length scales remains an exciting frontier of basic research in theoretical physics. A unifying theme in this area is the quantization of geometrical objects. The majority of lectures at the Advanced Study Institute on Quantum Ge ometry in Akureyri was on recent advances in superstring theory, which is the leading candidate for a unified description of all known elementary par ticles and interactions. The geometric concept of one-dimensional extended objects, or strings, has always been at the core of superstring theory but in recent years the focus has shifted to include also higher-dimensional ob jects, so called D-branes, which play a key role in the non-perturbative dynamics of the theory. A related development has seen the strong coupling regime of a given string theory identified with the weak coupling regime of what was previ ously believed to be a different theory, and a web of such" dualities" that interrelates all known superstring theories has emerged. The resulting uni fied theoretical framework, termed M-theory, has evolved at a rapid pace in recent years.

This book is the first volume of proceedings from the joint conference X International Symposium “Quantum Theory and Symmetries” (QTS-X) and XII International Workshop “Lie Theory and Its Applications in Physics” (LT-XII), held on 19–25 June 2017 in Varna, Bulgaria. The QTS series was founded on the core principle that symmetries underlie all descriptions of quantum systems. It has since evolved into a symposium at the forefront of theoretical and mathematical physics. The LT series covers the whole field of Lie theory in its widest sense, together with its applications in many areas of physics. As an interface between mathematics and physics, the workshop serves as a meeting place for mathematicians and theoretical and mathematical physicists. In dividing the material between the two volumes, the Editor has sought to select papers that are more oriented toward mathematics for the first volume, and those focusing more on physics for the second. However, this division is relative, since many papers are equally suitable for either volume. The topics addressed in this volume represent the latest trends in the fields covered by the joint conferences: representation theory, integrability, entanglement, quantum groups, number theory, conformal geometry, quantum affine superalgebras, noncommutative geometry. Further, they present various mathematical results: on minuscule modules, symmetry breaking operators, Kashiwara crystals, meta-conformal invariance, the superintegrable Zernike system.

Although ideas from quantum physics play an important role in many parts of modern mathematics, there are few books about quantum mechanics aimed at mathematicians. This book introduces the main ideas of quantum mechanics in language familiar to mathematicians. Readers with little prior exposure to physics will enjoy the book's conversational tone as they delve into such topics as the Hilbert space approach to quantum theory; the Schrödinger equation in one space dimension; the Spectral Theorem for bounded and unbounded self-adjoint operators; the Stone–von Neumann Theorem; the Wentzel–Kramers–Brillouin approximation; the role of Lie groups and Lie algebras in quantum mechanics; and the path-integral approach to quantum mechanics. The numerous exercises at the end of each chapter make the book suitable for both graduate courses and independent study. Most of the text is accessible to graduate students in mathematics who have had a first course in real analysis, covering the basics of L2 spaces and Hilbert spaces. The final chapters introduce readers who are familiar with the theory of manifolds to more advanced topics, including geometric quantization.

The author does not want a book description on the back cover.

The first title in a new series, this book explores topics from classical and quantum mechanics and field theory. The material is presented at a level between that of a textbook and research papers making it ideal for graduate students. The book provides an entree into a field that promises to remain exciting and important for years to come.

From the reviews: "...useful for experts in mathematical physics...this is a very interesting book, which deserves to be found in any physical library." (OPTICS & PHOTONICS NEWS, July/August 2005).

This book provides a comprehensive account of a modern generalisation of differential geometry in which coordinates need not commute. This requires a reinvention of differential geometry that refers only to the coordinate algebra, now possibly noncommutative, rather than to actual points. Such a theory is needed for the geometry of Hopf algebras or quantum groups, which provide key examples, as well as in physics to model quantum gravity effects in the form of quantum spacetime. The mathematical formalism can be applied to any algebra and includes graph geometry and a Lie theory of finite groups. Even the algebra of 2 x 2 matrices turns out to admit a rich moduli of quantum Riemannian geometries. The approach taken is a `bottom up’ one in which the different layers of geometry are built up in succession, starting from differential forms and proceeding up to the notion of a quantum `Levi-Civita’ bimodule connection, geometric Laplacians and, in some cases, Dirac operators. The book also covers elements of Connes’ approach to the subject coming from cyclic cohomology and spectral triples. Other topics include various other cohomology theories, holomorphic structures and noncommutative D-modules. A unique feature of the book is its constructive approach and its wealth of examples drawn from a large body of literature in mathematical physics, now put on a firm algebraic footing. Including exercises with solutions, it can be used as a textbook for advanced courses as well as a reference for researchers.

This book collects independent contributions on current developments in quantum information theory, a very interdisciplinary field at the intersection of physics, computer science and mathematics. Making intense use of the most advanced concepts from each discipline, the authors give in each contribution pedagogical introductions to the main concepts underlying their present research and present a personal perspective on some of the most exciting open problems. Keeping this diverse audience in mind, special efforts have been made to ensure that the basic concepts underlying quantum information are covered in an understandable way for mathematical readers, who can find there new open challenges for their research. At the same time, the volume can also be of use to physicists wishing to learn advanced mathematical tools, especially of differential and algebraic geometric nature.

The present work is the first volume of a substantially enlarged version of the mimeographed notes of a course of lectures first given by me in the Indian Statistical Institute, Calcutta, India, during 1964-65. When it was suggested that these lectures be developed into a book, I readily agreed and took the opportunity to extend the scope of the material covered. No background in physics is in principle necessary for understand ing the essential ideas in this work. However, a high degree of mathematical maturity is certainly indispensable. It is safe to say that I aim at an audience composed of professional mathematicians, advanced graduate students, and, hopefully, the rapidly increasing group of mathematical physicists who are attracted to fundamental mathematical questions. Over the years, the mathematics of quantum theory has become more abstract and, consequently, simpler. Hilbert spaces have been used from the very beginning and, after Weyl and Wigner, group representations have come in conclusively. Recent discoveries seem to indicate that the role of group representations is destined for further expansion, not to speak of the impact of the theory of several complex variables and function-space analysis. But all of this pertains to the world of interacting subatomic particles; the more modest view of the microscopic world presented in this book requires somewhat less. The reader with a knowledge of abstract integration, Hilbert space theory, and topological groups will find the going easy.

Graduate-level text develops group theory relevant to physics and chemistry and illustrates their applications to quantum mechanics, with systematic treatment of quantum theory of atoms, molecules, solids. 1964 edition.

Available for the first time in soft cover, this book is a classic on the foundations of quantum theory. It examines the subject from a point of view that goes back to Heisenberg and Dirac and whose definitive mathematical formulation is due to von Neumann. This view leads most naturally to the fundamental questions that are at the basis of all attempts to understand the world of atomic and subatomic particles.

This book presents the text of most of the lectures which were de livered at the Meeting Quantum Theories and Geometry which was held at the Fondation Les Treilles from March 23 to March 27, 1987. The general aim of this meeting was to bring together mathemati cians and physicists who have worked in this growing field of contact between the two disciplines, namely this region where geometry and physics interact creatively in both directions. It 1S the strong belief of the organizers that these written con tributions will be a useful document for research people workin~ 1n geometry or physics. Three lectures were devoted to the deformation approach to quantum mechanics which involves a modification of both the associative and the Lie structure of the algebra of functions on classical phase space. A. Lichnerowicz shows how one can view classical and quantum statistical mechanics in terms of a deformation with a parameter inversely propor tional to temperature. S. Gutt reviews the physical background of star products and indicates their applications in Lie groups representa tion theory and in harmonic analysis. D. Arnal gives a rigorous theory Vll viii PREFACI of the star exponential in the case of the Heisenberg group and shows how this can be extended to arbitrary nilpotent groups.

This book offers a complete discussion of techniques and topics intervening in the mathematical treatment of quantum and semi-classical mechanics. It starts with a very readable introduction to symplectic geometry. Many topics are also of genuine interest for pure mathematicians working in geometry and topology.

This book gives a detailed and self-contained introduction into the theory of spectral functions, with an emphasis on their applications to quantum field theory. All methods are illustrated with applications to specific physical problems from the forefront of current research, such as finite-temperature field theory, D-branes, quantum solitons and noncommutativity. In the first part of the book, necessary background information on differential geometry and quantization, including less standard material, is collected. The second part of the book contains a detailed description of main spectral functions and methods of their calculation. In the third part, the theory is applied to several examples (D-branes, quantum solitons, anomalies, noncommutativity). This book addresses advanced graduate students and researchers in mathematical physics with basic knowledge of quantum field theory and differential geometry. The aim is to prepare readers to use spectral functions in their own research, in particular in relation to heat kernels and zeta functions.

Mathematical Foundations of Quantum Theory is a collection of papers presented at the 1977 conference on the Mathematical Foundations of Quantum Theory, held in New Orleans. The contributors present their topics from a wide variety of backgrounds and specialization, but all shared a common interest in answering quantum issues. Organized into 20 chapters, this book's opening chapters establish a sound mathematical basis for quantum theory and a mode of observation in the double slit experiment. This book then describes the Lorentz particle system and other mathematical structures with which fundamental quantum theory must deal, and then some unsolved problems in the quantum logic approach to the foundations of quantum mechanics are considered. Considerable chapters cover topics on manuals and logics for quantum mechanics. This book also examines the problems in quantum logic, and then presents examples of their interpretation and relevance to nonclassical logic and statistics. The accommodation of conventional Fermi-Dirac and Bose-Einstein statistics in quantum mechanics or quantum field theory is illustrated. The final chapters of the book present a system of axioms for nonrelativistic quantum mechanics, with particular emphasis on the role of density operators as states. Specific connections of this theory with other formulations of quantum theory are also considered. These chapters also deal with the determination of the state of an elementary quantum mechanical system by the associated position and momentum distribution. This book is of value to physicists, mathematicians, and researchers who are interested in quantum theory.

This textbook is mainly for physics students at the advanced undergraduate and beginning graduate levels, especially those with a theoretical inclination. Its chief purpose is to give a systematic introduction to the main ingredients of the fundamentals of quantum theory, with special emphasis on those aspects of group theory (spacetime and permutational symmetries and group representations) and differential geometry (geometrical phases, topological quantum numbers, and Chern–Simons Theory) that are relevant in modern developments of the subject. It will provide students with an overview of key elements of the theory, as well as a solid preparation in calculational techniques.

Several well-established geometric and topological methods are used in this work in an application to a beautiful physical phenomenon known as the geometric phase. This book examines the geometric phase, bringing together different physical phenomena under a unified mathematical scheme. The material is presented so that graduate students and researchers in applied mathematics and physics with an understanding of classical and quantum mechanics can handle the text.

This text systematically presents the basics of quantum mechanics, emphasizing the role of Lie groups, Lie algebras, and their unitary representations. The mathematical structure of the subject is brought to the fore, intentionally avoiding significant overlap with material from standard physics courses in quantum mechanics and quantum field theory. The level of presentation is attractive to mathematics students looking to learn about both quantum mechanics and representation theory, while also appealing to physics students who would like to know more about the mathematics underlying the subject. This text showcases the numerous differences between typical mathematical and physical treatments of the subject. The latter portions of the book focus on central mathematical objects that occur in the Standard Model of particle physics, underlining the deep and intimate connections between mathematics and the physical world. While an elementary physics course of some kind would be helpful to the reader, no specific background in physics is assumed, making this book accessible to students with a grounding in multivariable calculus and linear algebra. Many exercises are provided to develop the reader's understanding of and facility in quantum-theoretical concepts and calculations.

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