

## Geometry Form G Chapter 5

Elements of Noncommutative Geometry Springer Science & Business Media Manifolds and Differential Geometry American Mathematical Soc.

This book offers a complete discussion of techniques and topics intervening in the mathematical treatment of quantum and semi-classical mechanics. It starts with a very readable introduction to symplectic geometry. Many topics are also of genuine interest for pure mathematicians working in geometry and topology.

Starting from classical arithmetical questions on quadratic forms, this book takes the reader step by step through the connections with lattice sphere packing and covering problems. As a model for polyhedral reduction theories of positive definite quadratic forms, Minkowski's classical theory is presented, including an application to multidimensional continued fraction expansions. The reduction theories of Voronoi are described in great detail, including full proofs, new views, and generalizations that cannot be found elsewhere. Based on Voronoi's second reduction theory, the local analysis of sphere coverings and several of its applications are presented. These include the classification of totally real thin number fields, connections to the Minkowski conjecture, and the discovery of new, sometimes surprising, properties of exceptional structures such as the Leech lattice or the root lattices. Throughout this book, special attention is paid to algorithms and computability, allowing computer-assisted treatments. Although dealing with relatively classical topics that have been worked on extensively by numerous authors, this book is exemplary in showing how computers may help to gain new insights.

This carefully written book is an introduction to the beautiful ideas and results of differential geometry. The first half covers the geometry of curves and surfaces, which provide much of the motivation and intuition for the general theory. The second part studies the geometry of general manifolds, with particular emphasis on connections and curvature. The text is illustrated with many figures and examples. The prerequisites are undergraduate analysis and linear algebra. This new edition provides many advancements, including more figures and exercises, and--as a new feature--a good number of solutions to selected exercises. The classical theory of stochastic processes has important applications arising from the need to describe irreversible evolutions in classical mechanics; analogously quantum stochastic processes can be used to model the dynamics of irreversible quantum systems. Noncommutative, i.e. quantum, geometry provides a framework in which quantum stochastic structures can be explored. This book is the first to describe how these two mathematical constructions are related. In particular, key ideas of semigroups and complete positivity are combined to yield quantum dynamical semigroups (QDS). Sinha and Goswami also develop a general theory of Evans-Hudson dilation for both bounded and unbounded coefficients. The unique features of the book, including the interaction of QDS and quantum stochastic calculus with noncommutative geometry and a thorough discussion of this calculus with unbounded coefficients, will make it of interest to graduate students and researchers in functional analysis, probability and mathematical physics.

Like any books on a subject as vast as this, this book has to have a point-of-view to guide the selection of topics. Naber takes the view that the rekindled interest that mathematics and physics have shown in each other of late should be fostered, and that this is best accomplished by allowing them to cohabit. The book weaves together rudimentary notions from the classical gauge theory of physics with the topological and geometrical concepts that became the mathematical models of these notions. The reader is asked to join the author on some vague notion of what an electromagnetic field might be, to be willing to accept a few of the more elementary pronouncements of quantum mechanics, and to have a solid background in real analysis and linear algebra and some of the vocabulary of modern algebra. In return, the book offers an excursion that begins with the definition of a topological space and finds its way eventually to the moduli space of anti-self-dual  $SU(2)$  connections on  $S^4$  with instanton number  $-1$ .

The discovery of new algorithms for dealing with polynomial equations, and their implementation on fast, inexpensive computers, has revolutionized algebraic geometry and led to exciting new applications in the field. This book details many uses of algebraic geometry and highlights recent applications of Grobner bases and resultants. This edition contains two new sections, a new chapter, updated references and many minor improvements throughout.

In this book we study sprays and Finsler metrics. Roughly speaking, a spray on a manifold consists of compatible systems of second-order ordinary differential equations. A Finsler metric on a manifold is a family of norms in tangent spaces, which vary smoothly with the base point. Every Finsler metric determines a spray by its systems of geodesic equations. Thus, Finsler spaces can be viewed as special spray spaces. On the other hand, every Finsler metric defines a distance function by the length of minimal curves. Thus Finsler spaces can be viewed as regular metric spaces. Riemannian spaces are special regular metric spaces. In 1854, B. Riemann introduced the Riemann curvature for Riemannian spaces in his ground-breaking Habilitationvortrag. Thereafter the geometry of these special regular metric spaces is named after him. Riemann also mentioned general regular metric spaces, but he thought that there were nothing new in the general case. In fact, it is technically much more difficult to deal with general regular metric spaces. For more than half century, there had been no essential progress in this direction until P. Finsler did his pioneering work in 1918. Finsler studied the variational problems of curves and surfaces in general regular metric spaces. Some difficult problems were solved by him. Since then, such regular metric spaces are called Finsler spaces. Finsler, however, did not go any further to introduce curvatures for regular metric spaces. He switched his research direction to set theory shortly after his graduation.

Differential geometry plays an increasingly important role in modern theoretical physics and applied mathematics. This textbook gives an introduction to geometrical topics useful in theoretical physics and applied mathematics, covering: manifolds, tensor fields, differential forms, connections, symplectic geometry, actions of Lie groups, bundles, spinors, and so on. Written in an informal style, the author places a strong emphasis on developing the understanding of the general theory through more than 1000 simple exercises, with complete solutions or detailed hints. The book will prepare readers for studying modern treatments of Lagrangian and Hamiltonian mechanics, electromagnetism, gauge fields, relativity and gravitation. Differential Geometry and Lie Groups for Physicists is well suited for courses in physics, mathematics

and engineering for advanced undergraduate or graduate students, and can also be used for active self-study. The required mathematical background knowledge does not go beyond the level of standard introductory undergraduate mathematics courses.

This unique two-volume set presents the subjects of stochastic processes, information theory, and Lie groups in a unified setting, thereby building bridges between fields that are rarely studied by the same people. Unlike the many excellent formal treatments available for each of these subjects individually, the emphasis in both of these volumes is on the use of stochastic, geometric, and group-theoretic concepts in the modeling of physical phenomena. Stochastic Models, Information Theory, and Lie Groups will be of interest to advanced undergraduate and graduate students, researchers, and practitioners working in applied mathematics, the physical sciences, and engineering. Extensive exercises, motivating examples, and real-world applications make the work suitable as a textbook for use in courses that emphasize applied stochastic processes or differential geometry.

Applies variational methods and critical point theory on infinite dimensional manifolds to some problems in Lorentzian geometry which have a variational nature, such as existence and multiplicity results on geodesics and relations between such geodesics and the topology of the manifold.

The title of this book is no surprise for people working in the field of Analytical Mechanics. However, the geometric concepts of Lagrange space and Hamilton space are completely new. The geometry of Lagrange spaces, introduced and studied in [76],[96], was extensively examined in the last two decades by geometers and physicists from Canada, Germany, Hungary, Italy, Japan, Romania, Russia and U.S.A. Many international conferences were devoted to debate this subject, proceedings and monographs were published [10], [18], [112], [113],... A large area of applicability of this geometry is suggested by the connections to Biology, Mechanics, and Physics and also by its general setting as a generalization of Finsler and Riemannian geometries. The concept of Hamilton space, introduced in [105], [101] was intensively studied in [63], [66], [97],... and it has been successful, as a geometric theory of the Hamiltonian function the fundamental entity in Mechanics and Physics. The classical Legendre's duality makes possible a natural connection between Lagrange and Hamilton spaces. It reveals new concepts and geometrical objects of Hamilton spaces that are dual to those which are similar in Lagrange spaces. Following this duality Cartan spaces introduced and studied in [98], [99],..., are, roughly speaking, the Legendre duals of certain Finsler spaces [98], [66], [67]. The above arguments make this monograph a continuation of [106], [113], emphasizing the Hamilton geometry.

This volume presents selected papers resulting from the meeting at Sundance on enumerative algebraic geometry. The papers are original research articles and concentrate on the underlying geometry of the subject.

This book gives an introduction to the basics of differential geometry, keeping in mind the natural origin of many geometrical quantities, as well as the applications of differential geometry and its methods to other sciences. The book is based on lectures the author held repeatedly at Novosibirsk State University. It is addressed to students as well as to anyone who wants to learn the basics of differential geometry.

Barsotti Symposium in Algebraic Geometry contains papers corresponding to the lectures given at the 1991 memorial meeting held in Abano Terme in honor of Iacopo Barsotti. This text reflects Barsotti's significant contributions in the field. This book is composed of 10 chapters and begins with a review of the centers of three-dimensional skylanin algebras. The succeeding chapters deal with the theoretical aspects of the Abelian varieties, Witt realization of p-Adic Barsotti-Tate Groups, and hypergeometric series and functions. These topics are followed by discussions of logarithmic spaces and the estimates for and inequalities among A-numbers. The closing chapter describes the moduli of Abelian varieties in positive characteristic. This book will be of value to mathematicians.

From the reviews: "... focused mainly on complex differential geometry and holomorphic bundle theory. This is a powerful book, written by a very distinguished contributor to the field" (Contemporary Physics ) "the book provides a large amount of background for current research across a spectrum of field. ... requires effort to read but it is worthwhile and rewarding" (New Zealand Math. Soc. Newsletter) " The contents are highly technical and the pace of the exposition is quite fast. Manin is an outstanding mathematician, and writer as well, perfectly at ease in the most abstract and complex situation. With such a guide the reader will be generously rewarded!" (Physicalia) This new edition includes an Appendix on developments of the last 10 years, by S. Merkulov.

The present book is intended as a textbook and reference work on three topics in the title. Together with a volume in progress on "Groups and Geometric Analysis" it supersedes my "Differential Geometry and Symmetric Spaces," published in 1962. Since that time several branches of the subject, particularly the function theory on symmetric spaces, have developed substantially. I felt that an expanded treatment might now be useful.

An introduction to geometrical topics used in applied mathematics and theoretical physics.

This book deals with such important subjects as variational methods, the continuity method, parabolic equations on fiber bundles, ideas concerning points of concentration, blowing-up technique, geometric and topological methods. It explores important geometric problems that are of interest to many mathematicians and scientists but have only recently been partially solved.

The present monograph is motivated by two distinct aims. Firstly, an endeavour has been made to furnish a reasonably comprehensive account of the theory of Finsler spaces based on the methods of classical differential geometry. Secondly, it is hoped that this monograph may serve also as an introduction to a branch of differential geometry which is closely related to various topics in theoretical physics, notably analytical dynamics and geometrical optics. With this second object in mind, an attempt has been made to describe the basic aspects of the theory in some detail - even at the expense of conciseness - while in the more specialised sections of the later chapters, which might be of interest chiefly to the specialist, a more succinct style has been adopted. The fact that there exist several fundamentally different points of view with regard to Finsler geometry has rendered the task of writing a coherent account a rather difficult one. This remark is relevant not only to the development of the subject on the basis of the tensor calculus, but is applicable in an even wider sense. The extensive work of H. BUSEMANN has opened up new avenues of approach to Finsler geometry which are independent of the methods of classical tensor analysis. In the latter sense, therefore, a full description of this approach does not fall within the scope of this treatise, although its fundamental significance cannot be doubted.

These notes are based on a course entitled "Symplectic Geometry and Geometric Quantization" taught by Alan Weinstein at the University of California, Berkeley (fall 1992) and at the Centre Emile Borel (spring 1994). The only prerequisite for the course needed is a knowledge of the basic notions from the theory of differentiable manifolds (differential forms, vector fields, transversality, etc.). The aim is to give students an introduction to the ideas of microlocal analysis and the related symplectic geometry, with an emphasis on the role these ideas play in formalizing the transition between the mathematics of classical dynamics (hamiltonian flows on symplectic manifolds) and quantum mechanics (unitary flows on Hilbert spaces). These notes are meant to function as a guide to the literature. The authors refer to other sources for many details that are omitted and can be bypassed on a first reading.

Geometry of Manifolds

This text is intended to serve as an introduction to the geometry of the action of discrete groups of Mobius transformations. The subject

matter has now been studied with changing points of emphasis for over a hundred years, the most recent developments being connected with the theory of 3-manifolds: see, for example, the papers of Poincare [77] and Thurston [101]. About 1940, the now well-known (but virtually unobtainable) Fenchel-Nielsen manuscript appeared. Sadly, the manuscript never appeared in print, and this more modest text attempts to display at least some of the beautiful geometrical ideas to be found in that manuscript, as well as some more recent material. The text has been written with the conviction that geometrical explanations are essential for a full understanding of the material and that however simple a matrix proof might seem, a geometric proof is almost certainly more profitable. Further, wherever possible, results should be stated in a form that is invariant under conjugation, thus making the intrinsic nature of the result more apparent. Despite the fact that the subject matter is concerned with groups of isometries of hyperbolic geometry, many publications rely on Euclidean estimates and geometry. However, the recent developments have again emphasized the need for hyperbolic geometry, and I have included a comprehensive chapter on analytical (not axiomatic) hyperbolic geometry. It is hoped that this chapter will serve as a "dictionary" of formulae in plane hyperbolic geometry and as such will be of interest and use in its own right.

Shafarevich's Basic Algebraic Geometry has been a classic and universally used introduction to the subject since its first appearance over 40 years ago. As the translator writes in a prefatory note, "For all [advanced undergraduate and beginning graduate] students, and for the many specialists in other branches of math who need a liberal education in algebraic geometry, Shafarevich's book is a must." The second volume is in two parts: Book II is a gentle cultural introduction to scheme theory, with the first aim of putting abstract algebraic varieties on a firm foundation; a second aim is to introduce Hilbert schemes and moduli spaces, that serve as parameter spaces for other geometric constructions. Book III discusses complex manifolds and their relation with algebraic varieties, Kähler geometry and Hodge theory. The final section raises an important problem in uniformising higher dimensional varieties that has been widely studied as the "Shafarevich conjecture". The style of Basic Algebraic Geometry 2 and its minimal prerequisites make it to a large extent independent of Basic Algebraic Geometry 1, and accessible to beginning graduate students in mathematics and in theoretical physics.

Adopting an elegant geometrical approach, this advanced pedagogical text describes deep and intuitive methods for understanding the subtle logic of supersymmetry while avoiding lengthy computations. The book describes how complex results and formulae obtained using other approaches can be significantly simplified when translated to a geometric setting. Introductory chapters describe geometric structures in field theory in the general case, while detailed later chapters address specific structures such as parallel tensor fields, G-structures, and isometry groups. The relationship between structures in supergravity and periodic maps of algebraic manifolds, Kodaira-Spencer theory, modularity, and the arithmetic properties of supergravity are also addressed. Relevant geometric concepts are introduced and described in detail, providing a self-contained toolkit of useful techniques, formulae and constructions. Covering all the material necessary for the application of supersymmetric field theories to fundamental physical questions, this is an outstanding resource for graduate students and researchers in theoretical physics.

Comprehensive coverage of the foundations, applications, recent developments, and future of conformal differential geometry Conformal Differential Geometry and Its Generalizations is the first and only text that systematically presents

the foundations and manifestations of conformal differential geometry. It offers the first unified presentation of the subject, which was established more than a century ago. The text is divided into seven chapters, each containing figures, formulas, and historical and bibliographical notes, while numerous examples elucidate the necessary theory. Clear, focused, and expertly synthesized, *Conformal Differential Geometry and Its Generalizations* \* Develops the theory of hypersurfaces and submanifolds of any dimension of conformal and pseudoconformal spaces. \* Investigates conformal and pseudoconformal structures on a manifold of arbitrary dimension, derives their structure equations, and explores their tensor of conformal curvature. \* Analyzes the real theory of four-dimensional conformal structures of all possible signatures. \* Considers the analytic and differential geometry of Grassmann and almost Grassmann structures. \* Draws connections between almost Grassmann structures and web theory. Conformal differential geometry, a part of classical differential geometry, was founded at the turn of the century and gave rise to the study of conformal and almost Grassmann structures in later years. Until now, no book has offered a systematic presentation of the multidimensional conformal differential geometry and the conformal and almost Grassmann structures. After years of intense research at their respective universities and at the Soviet School of Differential Geometry, Maks A. Akivis and Vladislav V. Goldberg have written this well-conceived, expertly executed volume to fill a void in the literature. Dr. Akivis and Dr. Goldberg supply a deep foundation, applications, numerous examples, and recent developments in the field. Many of the findings that fill these pages are published here for the first time, and previously published results are reexamined in a unified context. The geometry and theory of conformal and pseudoconformal spaces of arbitrary dimension, as well as the theory of Grassmann and almost Grassmann structures, are discussed and analyzed in detail. The topics covered not only advance the subject itself, but pose important questions for future investigations. This exhaustive, groundbreaking text combines the classical results and recent developments and findings. This volume is intended for graduate students and researchers of differential geometry. It can be especially useful to those students and researchers who are interested in conformal and Grassmann differential geometry and their applications to theoretical physics.

It is known that to any Riemannian manifold  $(M, g)$ , with or without boundary, one can associate certain fundamental objects. Among them are the Laplace-Beltrami operator and the Hodge-de Rham operators, which are natural [that is, they commute with the isometries of  $(M, g)$ ], elliptic, self-adjoint second order differential operators acting on the space of real valued smooth functions on  $M$  and the spaces of smooth differential forms on  $M$ , respectively. If  $M$  is closed, the spectrum of each such operator is an infinite divergent sequence of real numbers, each eigenvalue being repeated according to its finite multiplicity. Spectral Geometry is concerned with the spectra of these operators, also the extent to which these spectra determine the geometry of  $(M, g)$  and the topology of  $M$ . This problem has been translated by

several authors (most notably M. Kac). into the colloquial question "Can one hear the shape of a manifold?" because of its analogy with the wave equation. This terminology was inspired from earlier results of H. Weyl. It is known that the above spectra cannot completely determine either the geometry of  $(M, g)$  or the topology of  $M$ . For instance, there are examples of pairs of closed Riemannian manifolds with the same spectra corresponding to the Laplace-Beltrami operators, but which differ substantially in their geometry and which are even not homotopically equivalent.

The purpose of this handbook is to give an overview of some recent developments in differential geometry related to supersymmetric field theories. The main themes covered are: Special geometry and supersymmetry Generalized geometry Geometries with torsion Para-geometries Holonomy theory Symmetric spaces and spaces of constant curvature Conformal geometry Wave equations on Lorentzian manifolds D-branes and K-theory The intended audience consists of advanced students and researchers working in differential geometry, string theory, and related areas. The emphasis is on geometrical structures occurring on target spaces of supersymmetric field theories. Some of these structures can be fully described in the classical framework of pseudo-Riemannian geometry. Others lead to new concepts relating various fields of research, such as special Kahler geometry or generalized geometry.

'Et moi ..., si j'avait su comment en revenir, One service mathematics has rendered the je n'y serais point aile.' human race. It has put common sense back Jules Verne where it belongs, on the topmost shelf next to the dusty canister labelled 'discarded n- sense'. The series is divergent; therefore we may be able to do something with it. Eric T. Bell O. Heaviside Mathematics is a tool for thought. A highly necessary tool in a world where both feedback and non linearities abound. Similarly, all kinds of parts of mathematics serve as tools for other parts and for other sciences. Applying a simple rewriting rule to the quote on the right above one finds such statements as: 'One service topology has rendered mathematical physics ...'; 'One service logic has rendered computer science ...'; 'One service category theory has rendered mathematics ...'. All arguably true. And all statements obtainable this way form part of the raison d'etre of this series.

This volume contains 17 surveys that cover many recent developments in Discrete Geometry and related fields. Besides presenting the state-of-the-art of classical research subjects like packing and covering, it also offers an introduction to new topological, algebraic and computational methods in this very active research field. The readers will find a variety of modern topics and many fascinating open problems that may serve as starting points for research.

The geometry of Jordan and Lie structures tries to answer the following question: what is the integrated, or geometric, version of real Jordan algebras, - triple systems and - pairs? Lie theory shows the way one has to go: Lie groups and symmetric spaces are the geometric version of Lie algebras and Lie triple systems. It turns out that both geometries are

closely related via a functor between them, called the Jordan-Lie functor, which is constructed in this book. The reader is not assumed to have any knowledge of Jordan theory; the text can serve as a self-contained introduction to (real finite-dimensional) Jordan theory.

Differential geometry began as the study of curves and surfaces using the methods of calculus. In time, the notions of curve and surface were generalized along with associated notions such as length, volume, and curvature. At the same time the topic has become closely allied with developments in topology. The basic object is a smooth manifold, to which some extra structure has been attached, such as a Riemannian metric, a symplectic form, a distinguished group of symmetries, or a connection on the tangent bundle. This book is a graduate-level introduction to the tools and structures of modern differential geometry. Included are the topics usually found in a course on differentiable manifolds, such as vector bundles, tensors, differential forms, de Rham cohomology, the Frobenius theorem and basic Lie group theory. The book also contains material on the general theory of connections on vector bundles and an in-depth chapter on semi-Riemannian geometry that covers basic material about Riemannian manifolds and Lorentz manifolds. An unusual feature of the book is the inclusion of an early chapter on the differential geometry of hyper-surfaces in Euclidean space. There is also a section that derives the exterior calculus version of Maxwell's equations. The first chapters of the book are suitable for a one-semester course on manifolds. There is more than enough material for a year-long course on manifolds and geometry.

Offering some of the topics of contemporary mathematical research, this fourth edition includes a systematic introduction to Kahler geometry and the presentation of additional techniques from geometric analysis.

Differential Geometry from a Singularity Theory Viewpoint provides a new look at the fascinating and classical subject of the differential geometry of surfaces in Euclidean spaces. The book uses singularity theory to capture some key geometric features of surfaces. It describes the theory of contact and its link with the theory of caustics and wavefronts. It then uses the powerful techniques of these theories to deduce geometric information about surfaces embedded in 3, 4 and 5-dimensional Euclidean spaces. The book also includes recent work of the authors and their collaborators on the geometry of sub-manifolds in Minkowski spaces. Contents: The Case for the Singularity Theory Approach Submanifolds of the Euclidean Space Singularities of Germs of Smooth Mappings Contact Between Submanifolds of  $n$ -Lagrangian and Legendrian Singularities Surfaces in the Euclidean 3-Space Surfaces in the Euclidean 4-Space Surfaces in the Euclidean 5-Space Spacelike Surfaces in the Minkowski Space-Time Global Viewpoint Readership: Advanced undergraduates and post-graduate students, and researchers in the fields of differential geometry and singularity theory. Key Features: The book is unique in its nature. It provides a coherent approach for studying the geometry of sub-manifolds of various ambient spaces from the singularity theory point of view. The book informs the reader about the progress in the field of extrinsic differential geometry and singularity theory. The information is new and has not been treated in previous textbooks. The book gathers scattered work from various research articles, most of which are recent, and describes techniques that could be used to tackle problems in other areas of mathematics. Keywords: Contact; Extrinsic

Geometry;Genericity;Caustics;Singularities;Surfaces;Transversality;Wave Fronts

This book is divided into fourteen chapters, with 18 appendices as introduction to prerequisite topological and algebraic knowledge, etc. The first seven chapters focus on local analysis. This part can be used as a fundamental textbook for graduate students of theoretical physics. Chapters 8–10 discuss geometry on fibre bundles, which facilitates further reference for researchers. The last four chapters deal with the Atiyah-Singer index theorem, its generalization and its application, quantum anomaly, cohomology field theory and noncommutative geometry, giving the reader a glimpse of the frontier of current research in theoretical physics.

This monograph, which is the first to be devoted to the geometry of multidimensional three-webs, presents the classical and up-to-date results of the theory, and those parts of geometry and algebra which are closely connected with it. Many problems of the theory of smooth quasigroups and loops are considered. In addition to the general theory of webs, important classes of special webs are also studied. The volume contains eight chapters dealing with geometric and algebraic structures associated with three-webs, transversally geodesic and isoclinic three-webs, Bol and Moufang three-webs, closed G-structures, automorphisms of three-webs, the geometry of the fourth-order differential neighborhood of a multidimensional three-web, and d-webs of codimension  $r$ . The book concludes with some appendices and a comprehensive bibliography. This volume will be of particular interest to graduate students and researchers working in the areas of differential and algebraic geometry and algebra.

Covering an exciting and active area of research at the crossroads of several different fields in mathematics and physics, and drawing on the author's previous work, this text has been written to explain the advanced mathematics involved simply and clearly to graduate students in both disciplines.

"Finsler Geometry: An Approach via Randers Spaces" exclusively deals with a special class of Finsler metrics -- Randers metrics, which are defined as the sum of a Riemannian metric and a 1-form. Randers metrics derive from the research on General Relativity Theory and have been applied in many areas of the natural sciences. They can also be naturally deduced as the solution of the Zermelo navigation problem. The book provides readers not only with essential findings on Randers metrics but also the core ideas and methods which are useful in Finsler geometry. It will be of significant interest to researchers and practitioners working in Finsler geometry, even in differential geometry or related natural fields. Xinyue Cheng is a Professor at the School of Mathematics and Statistics of Chongqing University of Technology, China. Zhongmin Shen is a Professor at the Department of Mathematical Sciences of Indiana University Purdue University, USA.

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