Geometrical Foundations Of Continuum Mechanics An Application To First And Second Order Elasticity And Elasto Plasticity Lecture Notes In Applied Mathematics And Mechanics

This contributed volume explores the applications of various topics in modern differential geometry to the foundations of continuum mechanics. In particular, the contributors use notions from areas such as global analysis, algebraic topology, and geometric measure theory. Chapter authors are experts in their respective areas, and provide important insights from the most recent research. Organized into two parts, the book first covers kinematics, forces, and stress theory, and then addresses defects, uniformity, and homogeneity. Specific topics covered include: Global stress and hyper-stress theories Applications of de Rham currents to singular dislocations Manifolds of mappings for continuum mechanics Kinematics of defects in solid crystals Geometric Continuum Mechanics will appeal to graduate students and researchers in the fields of mechanics, physics, and engineering who seek a more rigorous mathematical understanding of the area. Mathematicians interested in applications of analysis and geometry will also find the topics covered here of interest.

A concise introductory course text on continuum mechanics Fundamentals of Continuum Mechanics focuses on the fundamentals of the subject and provides the background for formulation of numerical methods for large deformations and a wide range of material behaviours. It aims to provide the foundations for further study, not just of these subjects, but also the formulations for much more complex material behaviour and their implementation computationally. This book is divided into 5 parts, covering mathematical preliminaries, stress, motion and deformation, balance of mass, momentum and energy, and ideal constitutive relations and is a suitable textbook for introductory graduate courses for students in mechanical and civil engineering, as well as those studying material science, geology and geophysics and biomechanics. A concise introductory course text on continuum mechanics Covers the fundamentals of continuum mechanics Uses modern tensor notation Contains problems and accompanied by a companion website hosting solutions Suitable as a textbook for introductory graduate courses for students in mechanical and civil engineering

Continuum mechanics studies the foundations of deformable body mechanics from a mathematical perspective. It also acts as a base upon which other applied areas such as solid mechanics and fluid mechanics are developed. This book discusses some important topics, which have come into prominence in the latter half of the twentieth century, such as material symmetry, frame-indifference and thermomechanics. The study begins with the necessary mathematical background in the form of an introduction to tensor analysis followed by a discussion on kinematics, which deals with purely geometrical notions such as strain and rate of deformation. Moving on to derivation of the governing equations, the book also presents applications in the areas of linear and nonlinear elasticity. In addition, the volume also provides a mathematical explanation to the axioms and laws of deformable body mechanics, and its various applications in the field of solid mechanics. This book illustrates the deep roots of the geometrically nonlinear kinematics of generalized continuum mechanics in differential geometry. Besides applications to first- order elasticity and elasto-plasticity an appreciation thereof is particularly illuminating for generalized models of continuum mechanics such as second-order (gradient-type) elasticity and elasto-plasticity. After a motivation that arises from considering geometrically linear first- and second- order crystal plasticity in Part I several concepts from differential geometry, relevant for what follows, such as connection, parallel transport, torsion, curvature, and metric for holonomic and anholonomic coordinate transformations are reiterated in Part II. Then, in Part III, the kinematics of geometrically nonlinear continuum mechanics are considered. There various concepts of differential geometry, in particular aspects related to compatibility, are generically applied to the kinematics of first- and second- order geometrically nonlinear continuum mechanics. Together with the discussion on the integrability conditions for the distortions and double-distortions, the concepts of dislocation, disclination and point-defect density tensors are introduced. For concreteness, after touching on nonlinear fir st- and second-order elasticity, a detailed discussion of the kinematics of (multiplicative) first- and second-order elasto-plasticity is given. The discussion naturally culminates in a comprehensive set of different types of dislocation, disclination and point-defect density tensors. It is argued, that these can potentially be used to model densities of geometrically necessary defects and the accompanying hardening in crystalline materials. Eventually Part IV summarizes the above findings on integrability whereby distinction is made between the straightforward conditions for the distortion and the double-distortion being integrable and the more involved conditions for the strain (metric) and the double-strain (connection) being integrable. The book addresses readers with an interest in continuum modelling of solids from engineering and the sciences alike, whereby a sound knowledge of tensor calculus and continuum mechanics is required as a prerequisite. .

This mathematically-oriented introduction takes the point of view that students should become familiar, at an early stage, with the physics of relativistic continua and thermodynamics within the framework of special relativity. Therefore, in addition to standard textbook topics such as relativistic kinematics and vacuum electrodynamics, the reader will be thoroughly introduced to relativistic continuum and fluid mechanics. There is emphasis on the 3+1 splitting technique.

Epstein presents the fundamental concepts of modern differential geometry within the framework of continuum mechanics. Divided into three parts of roughly equal length, the book opens with a motivational chapter to impress upon the reader that differential geometry is indeed the natural language of continuum mechanics or, better still, that the latter is a prime example of the application and materialisation of the former. In the second part, the fundamental notions of differential geometry are presented with rigor using a writing style that is as informal as possible. Differentiable manifolds, tangent bundles, exterior derivatives, Lie derivatives, and Lie groups are illustrated in terms of their mechanical interpretations. The third part includes the theory of fiber bundles, G-structures, and groupoids, which are applicable to bodies with internal structure and to the description of material inhomogeneity. The abstract notions of differential geometry are thus illuminated by practical and intuitively meaningful engineering applications.

This monograph provides a compendium of established and novel error estimation procedures applied in the field of Computational Mechanics. It also includes detailed derivations of these procedures to offer insights into the concepts used to control the errors obtained from employing Galerkin methods in finite and linearized hyperelasticity. The Galerkin methods introduced are considered advanced methods because they remedy certain shortcomings of the well-established finite element method, which is the archetypal Galerkin (mesh-based) method. In particular, this monograph focuses on the systematical derivation of the shape functions used to construct both Galerkin mesh-based and meshfree methods. The mesh-based methods considered are the (conventional) displacement-based, (dual-)mixed, smoothed, and extended finite element methods. In addition, it introduces the element-free Galerkin and reproducing kernel particle methods as representatives of a class of Galerkin meshfree methods. Including illustrative numerical examples relevant to engineering with an emphasis on elastic fracture mechanics problems, this monograph is intended for students, researchers, and practitioners aiming to increase the reliability of their numerical simulations and wanting to better grasp the concepts of Galerkin methods and associated error estimation procedures.

The foundations of the geometry of a continuum approximation to the deformation of a crystalline solid were investigated. A method of formulating the infinitesimal deformation of a system of particles based on an averaging process was developed. This formulation was based on the construction of a polyhedral mesh'' valid for any system of particles whether or not they lie in a lattice configuration. However, when the particles lie in a perfect lattice, the mesh is shown to yield the familiar definition of dislocation motion. The averaging process was then extended to include deformations at grain boundaries. Using these results, the concept of infinitesimal plastic transformation was formulated and it is shown that by assuming the initial state of the material to be described by a spatial affine connection, the entire dynamic description of the material deformation is then given by a four dimensional space-time affine connection whose invariants together with the integrated strain define the state of the material. Equations of continuity for plastic as well as for ordinary elastic deformation were derived. The exterior calculus of E. Cartan was utilized to simplify the computations. (auth).

This research monograph discusses novel approaches to geometric continuum mechanics and introduces beams as constraint continuous bodies. In the coordinate free and metric independent geometric formulation of continuum mechanics as well as for beam theories, the principle of virtual work serves as the fundamental principle of mechanics. Based on the perception of analytical mechanics that forces of a mechanical system are defined as dual quantities to the kinematical description, the virtual work approach is a systematic way to treat arbitrary mechanical systems. Whereas this methodology is very convenient to formulate induced beam theories, it is essential in geometric continuum mechanics when the assumptions on the physical space are relaxed and the space is modeled as a smooth manifold. The book addresses researcher and graduate students in engineering and mathematics interested in recent developments of a geometric formulation of continuum mechanics and a hierarchical development of induced beam theories.

Across the centuries, the development and growth of mathematical concepts have been strongly stimulated by the needs of mechanics. Vector algebra was developed to describe the equilibrium of force systems and originated from Stevin's experiments (1548-1620). Vector analysis was then introduced to study velocity fields and force fields. Classical dynamics required the differential calculus developed by Newton (1687). Nevertheless, the concept of particle acceleration was the starting point for introducing a structured spacetime. Instantaneous velocity involved the set of particle positions in space. Vector algebra theory was not sufficient to compare the different velocities of a particle in the course of time. There was a need to (parallel) transport these velocities at a single point before any vector algebraic operation. The appropriate mathematical structure for this transport was the connection. I The Euclidean connection derived from the metric tensor of the referential body was the only connection used in mechanics for over two centuries. Then, major steps in the evolution of spacetime concepts were made by Einstein in 1905 (special relativity) and 1915 (general relativity) by using Riemannian connection. Slightly later, nonrelativistic spacetime which includes the main features of general relativity I It took about one and a half centuries for connection theory to be accepted as an independent theory in mathematics. Major steps for the connection concept are attributed to a series of findings: Riemann 1854, Christoffel 1869, Ricci 1888, Levi-Civita 1917, WeyJ 1918, Cartan 1923, Eshermann 1950.

During the last decades, continuum mechanics of porous materials has achieved great attention, since it allows for the consideration of the volumetrically coupled behaviour of the solid matrix deformation and the pore-fluid flow. Naturally, applications of porous media models range from civil and environmental engineering, where, e. g. , geote- nical problems like the consolidation problem are of great interest, via mechanical engineering, where, e. g. , the description of sinter materials or polymeric and metallic foams is a typical problem, to chemical and biomechanical engineering, where, e. g. , the complex structure of l- ing tissues is studied. Although these applications are principally very different, they basically fall into the category of multiphase materials, which can be described, on the macroscale, within the framework of the well-founded Theory of Porous Media (TPM). With the increasing power of computer hardware together with the rapidly decreasing computational costs, numerical solutions of complex coupled problems became possible and have been seriously investigated. However, since the quality of the numerical solutions strongly depends on the quality of the underlying physical model together with the experimental and mathematical possibilities to successfully determine realistic

material parameters, a successful treatment of porous materials requires a joint consideration of continuum mechanics, experimental mechanics and numerical methods. In addition, micromechanical - vestigations and homogenization techniques are very helpful to increase the phenomenological understanding of such media.
This book is a comprehensive study of the life and mathematics of Walter Noll, who helped to create the mathematical tools of modern rational mechanics and thermodynamics. Noll is one of the brilliant mathematicians of the second part of the 20th century. His contribution is large in both the applied and pure mathematics. The book stresses particularly Noll's method of axiomatization of physical theories, his axiomatics of continuum mechanics, thermodynamics of materials, special relativity theory, his discovery of the neo-classical space-time of mechanics, his theories of inhomogeneities in simple bodies, fit regions, contact interactions, annihilators of linear differential operators, and finite-dimensional spaces. It is a must for every mathematician, physicist, engineer or graduate student as a reference and key to Noll's mathematical heritage.
Presents a self-contained introduction to continuum mechanics that illustrates how many of the important partial differential equations of applied mathematics arise from continuum modeling principles Written as an accessible introduction, Continuum Mechanics: The Birthplace of Mathematical Models provides a comprehensive foundation for mathematical models used in fluid mechanics, solid mechanics, and heat transfer. The book features derivations of commonly used differential equations based on the fundamental continuum mechanical concepts encountered in various fields, such as engineering, physics, and geophysics. The book begins with geometric, algebraic, and analytical foundations before introducing topics in kinematics. The book

then addresses balance laws, constitutive relations, and constitutive theory. Finally, the book presents an approach to multiconstituent continua based on mixture theory to illustrate how phenomena, such as diffusion and porous-media flow, obey continuum-mechanical principles. Continuum Mechanics: The Birthplace of Mathematical Models features: Direct vector and tensor notation to minimize the reliance on particular coordinate systems when presenting the theory Terminology that is aligned with standard courses in vector calculus and linear algebra The use of Cartesian coordinates in the examples and problems to provide readers with a familiar setting Over 200 exercises and problems with hints and solutions in an appendix Introductions to constitutive theory and multiconstituent continua, which are distinctive for books at this level Continuum Mechanics: The Birthplace of Mathematical Models is an ideal textbook for courses on continuum mechanics for upper-undergraduate mathematics majors and graduate students in applied mathematics, mechanical engineering, civil engineering, physics, and geophysics. The book is also an excellent reference for professional mathematicians, physical scientists, and engineers.

German scholars, against odds now not only forgotten but also hard to imagine, were striving to revivify the life of the mind which the mental and physical barbarity preached and practised by the -isms and -acies of 1933-1946 had all but eradicated. Thinking that among the disciples of these elders, restorers rather than progressives, I might find a student or two who would wish to master new mathematics but grasp it and use it with the wholeness of earlier times, in 1952 I wrote to Mr. HAMEL, one of the few then remaining mathematicians from the classical mould, to ask him to name some young men fit to study for the doc torate in The Graduate Institute for Applied Mathematics at Indiana University, flourishing at that time though soon to be destroyed by the jealous ambition of the local, stereotyped pure. Having just retired from the Technische Universitat in Charlottenburg, he passed my inquiry on to Mr. SZABO, in whose institute there NOLL was then an assistant. Although Mr.

Graduate-level study approaches mathematical foundations of three-dimensional elasticity using modern differential geometry and functional analysis. It presents a classical subject in a modern setting, with examples of newer mathematical contributions. 1983 edition.

This classic work gives an excellent overview of the subject, with an emphasis on clarity, explanation, and motivation. Extensive exercises and a valuable section containing hints and answers make this an excellent text for both classroom use and independent study.

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Ian Murdoch's Physical Foundations of Continuum Mechanics will interest engineers, mathematicians, and physicists who study the macroscopic behaviour of solids and fluids or engage in molecular dynamical simulations. In contrast to standard works on the subject, Murdoch's book examines physical assumptions implicit in continuum modelling from a molecular perspective. In so doing, physical interpretations of concepts and fields are clarified by emphasising both their microscopic origin and sensitivity to scales of length and time. Murdoch expertly applies this approach to theories of mixtures, generalised continua, fluid flow through porous media, and systems whose molecular content changes with time. Elements of statistical mechanics are included, for comparison, and two extensive appendices address relevant mathematical concepts and results. This unique and thorough work is an authoritative reference for both students and experts in the field.

This book gives a comprehensive account of the formulation and computational treatment of basic geometrically linear models in 1D. To set the stage, it assembles some preliminaries regarding necessary modelling, computational and mathematical tools. Thereafter, the remaining parts are concerned with the actual catalogue of computational material models. To this end, after starting out with elasticity as a reference, further 15 different basic variants of material models (5 x each of {visco-elasticity, plasticity, visco-plasticity}, respectively) are systematically explored. The presentation for each of these basic material models is a stand-alone account and follows in each case the same structure. On the one hand, this allows, in the true sense of a catalogue, to consult each of the basic material models separately without the need to refer to other basic material models. On the other hand, even though this somewhat repetitious concept may seem tedious, it allows to compare the formulation and resulting algorithmic setting of the various basic material models and thereby to uncover, in detail, similarities and differences. In particular, the response of each basic material model is analysed for the identical histories (Zig-Zag, Sine, Ramp) of prescribed strain and stress so as to clearly showcase and to contrast to each other the characteristics of the various modelling options.

This book is designed for students in engineering, physics and mathematics. The material can be taught from the beginning of the third academic year. It could also be used for self study, given its pedagogical structure and the numerous solved problems which prepare for modem physics and technology. One of the original aspects of this work is the development together of the basic theory of tensors and the foundations of continuum mechanics. Why two books in one? Firstly, Tensor Analysis provides a thorough introduction of intrinsic mathematical entities, called tensors, which is essential for continuum mechanics. This way of proceeding greatly unifies the various subjects. Only some basic knowledge of linear algebra is necessary to start out on the topic of tensors. The essence of the mathematical foundations is introduced in a practical way. Tensor developments are often too abstract, since they are either aimed at algebraists only, or too quickly applied to physicists and engineers. Here a good balance has been found which allows these extremes to be brought closer together. Though the exposition of tensor theory forms a subject in itself, it is viewed not only as an autonomous mathematical discipline, but as a preparation for theories of physics and engineering. More specifically, because this part of the work deals with tensors in general coordinates and not solely in Cartesian coordinates, it will greatly help with many different disciplines such as differential geometry, analytical mechanics, continuum mechanics, special relativity, general relativity, cosmology, electromagnetism, quantum mechanics, etc ..

This book presents an introduction to material theory and, in particular, to elasticity, plasticity and viscoelasticity, to bring the reader close to the frontiers of today's knowledge in these particular fields. It starts right from the beginning without assuming much knowledge of the subject. Hence, the book is generally comprehensible to all engineers, physicists, mathematicians, and others. At the beginning of each new section, a brief Comment on the Literature contains recommendations for further reading. This book includes an updated reference list and over 100 changes throughout the book. It contains the latest knowledge on the subject. Two new chapters have been added in this new edition. Now finite viscoelasticity is included, and an Essay on gradient materials, which have recently drawn much attention.

The 39 papers in this collection are devoted mostly to the exact mathematical analysis of problems in continuum mechanics, but also to problems of a purely mathematical nature mainly connected to partial differential equations from continuum physics. All the papers are dedicated to J. Serrin and were originally published in the "Archive of Rational Mechanics and Analysis".

This book examines the exciting interface between differential geometry and continuum mechanics, now recognised as being of increasing technological significance. Topics discussed include isometric embeddings in differential geometry and the relation with microstructure in nonlinear elasticity, the use of manifolds in the description of microstructure in continuum mechanics, experimental measurement of microstructure, defects, dislocations, surface energies, and nematic liquid crystals. Compensated compactness in partial differential equations is also treated. The volume is intended for specialists and non-specialists in pure and applied geometry, continuum mechanics, theoretical physics, materials and engineering sciences, and partial differential equations. It will also be of interest to postdoctoral scientists and advanced postgraduate research students. These proceedings include revised written versions of the majority of papers presented by leading experts at the ICMS Edinburgh Workshop on Differential Geometry and Continuum Mechanics held in June 2013. All papers have been peer reviewed.

This book describes thermoelastic and inelastic deformation processes in crystalline solids undergoing loading by shock compression. Constitutive models with a basis in geometrically nonlinear continuum mechanics supply these descriptions. Large deformations such as finite strains and rotations, are addressed. The book covers dominant mechanisms of nonlinear thermoelasticity, dislocation plasticity, deformation twinning, fracture, flow, and other structure changes. Rigorous derivations of theoretical results are provided, with approximately 1300 numbered equations and an extensive bibliography of over 500 historical and modern references spanning from the 1920s to the present day. Case studies contain property data, as well as analytical, and numerical solutions to shock compression problems for different materials. Such materials are metals, ceramics, and minerals, single crystalline and polycrystalline. The intended audience of this book is practicing scientists (physicists, engineers, materials scientists, and applied mathematicians) involved in advanced research on shock compression of solid materials.

This book illustrates the deep roots of the geometrically nonlinear kinematics of generalized continuum mechanics in differential geometry. Besides applications to first- order elasticity and elasto-plasticity an appreciation thereof is particularly illuminating for generalized models of continuum mechanics such as second-order (gradient-type) elasticity and elasto-plasticity. After a motivation that arises from considering geometrically linear first- and second- order crystal plasticity in Part I several concepts from differential geometry, relevant for what follows, such as connection, parallel transport, torsion, curvature, and metric for holonomic and anholonomic coordinate transformations are reiterated in Part II. Then, in Part III, the kinematics of geometrically nonlinear continuum mechanics are considered. There various concepts of differential

geometry, in particular aspects related to compatibility, are generically applied to the kinematics of first- and second- order geometrically nonlinear continuum mechanics. Together with the discussion on the integrability conditions for the distortions and double-distortions, the concepts of dislocation, disclination and point-defect density tensors are introduced. For concreteness, after touching on nonlinear fir st- and second-order elasticity, a detailed discussion of the kinematics of (multiplicative) first- and second-order elasto-plasticity is given. The discussion naturally culminates in a comprehensive set of different types of dislocation, disclination and point-defect density tensors. It is argued, that these can potentially be used to model densities of geometrically necessary defects and the accompanying hardening in crystalline materials. Eventually Part IV summarizes the above findings on integrability whereby distinction is made between the straightforward conditions for the distortion and the double-distortion being integrable and the more involved conditions for the strain (metric) and the double-strain (connection) being integrable. The book addresses readers with an interest in continuum modelling of solids from engineering and the sciences alike, whereby a sound knowledge of tensor calculus and continuum mechanics is required as a prerequisite.
This book provides a brief introduction to rational

continuum mechanics in a form suitable for students of engineering, mathematics and science. The presentation is tightly focused on the simplest case of the classical mechanics of nonpolar materials, leaving aside the effects of internal structure, temperature and electromagnetism, and excluding other mathematical models, such as statistical mechanics, relativistic mechanics and quantum mechanics. Within the limitations of the simplest mechanical theory, the author had provided a text that is largely self-contained. Though the book is primarily an introduction to continuum mechanics, the lure and attraction inherent in the subject may also recommend the book as a vehicle by which the student can obtain a broader appreciation of certain important methods and results from classical and modern analysis.

A First Course in Rational Continuum Mechanics V1

This second part of Continuum Thermodynamics is designed to match almost one-to-one the chapters of Part I. This is done so that the reader studying thermodynamics will have a deepened understanding of the subjects covered in Part I. The aims of the book are in particular: the illustration of basic features of some simple thermodynamical models such as ideal and viscous fluids, non-Newtonian fluids, nonlinear solids, interactions with electromagnetic fields and diffusive porous materials. A further aim is the illustration of the

above subjects by examples and simple solutions of initial and boundary problems as well as simple exercises to develop skills in the construction of interdisciplinary macroscopic models.
Differential Geometry offers a concise introduction to some basic notions of modern differential geometry and their applications to solid mechanics and physics. Concepts such as manifolds, groups, fibre bundles and groupoids are first introduced within a purely topological framework. They are shown to be relevant to the description of space-time, configuration spaces of mechanical systems, symmetries in general, microstructure and local and distant symmetries of the constitutive response of continuous media. Once these ideas have been grasped at the topological level, the differential structure needed for the description of physical fields is introduced in terms of differentiable manifolds and principal frame bundles. These mathematical concepts are then illustrated with examples from continuum kinematics, Lagrangian and Hamiltonian mechanics, Cauchy fluxes and dislocation theory. This book will be useful for researchers and graduate students in science and engineering.
Continuum Mechanics (CM) is a natural field of application of concepts and methods of Differential Geometry (DG). The very foundations of both disciplines are intertwined in a deep manner. A presentation of basic issues in CM adopting the

powerful tools of modern DG is still substantially lacking. This booklet is intended to contribute to fill this gap, with specific reference to Elasticity theory. The classical subject is thoroughly revisited and revised in its basic aspects and in the general context of finite deformations. A case study of rubber-like materials enlightens the new concepts introduced by the geometric theory and opens the way for applications to soft materials such as the ones of interest in biomechanics.
Foundations and Applications of Mechanics: Volume II, Fluid Mechanics shows how suitable approximations such as ideal fluid flow model, boundary layer methods, and the acoustic approximation, can help solve problems of practical importance. The author proceeds from the general to the particular, making it clear at each stage what assumptions have been made to obtain a particular approximation. In his discussion of compressible fluids, Jog steers away from using gas tables and emphasizes obtaining solutions by numerical techniques - an approach more amenable to computer solutions. He discusses the control volume and the differential equation forms of governing equations in detail and uses examples to demonstrate the advantages and shortcomings of each approach.