

# Financial Mathematics For Actuaries Chapter 10

This text covers life tables, survival models, and life insurance premiums and reserves. It presents the actuarial material conceptually with reference to ideas from other mathematical studies, allowing readers with knowledge in calculus to explore business, actuarial science, economics, and statistics. Each chapter contains exercise sets and worked examples, which highlight the most important and frequently used formulas and show how the ideas and formulas work together smoothly. Illustrations and solutions are also provided.

to Actuarial Mathematics by A. K. Gupta Bowling Green State University, Bowling Green, Ohio, U. S. A. and T. Varga National Pension Insurance Fund. Budapest, Hungary SPRINGER-SCIENCE+BUSINESS MEDIA, B. V. A C. I. P. Catalogue record for this book is available from the Library of Congress. ISBN 978-90-481-5949-9 ISBN 978-94-017-0711-4 (eBook) DOI 10.

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PREFACE . . . . . ix

CHAPTER 1. FINANCIAL MATHEMATICS . . . . . 1

1. 1. Compound Interest . . . . . 1

1. 2. Present Value. . . . . 31

1. 3. Annuities. . . . . 48

CHAPTER 2. MORTALITY . . . . . 80

2. 1 Survival Time . . . . . 80

2. 2. Actuarial Functions of Mortality. . . . . 84

2. 3. Mortality Tables. . . . . 98

CHAPTER 3. LIFE INSURANCES AND ANNUITIES . . . . . 112

3. 1. Stochastic Cash Flows . . . . . 112

3. 2. Pure Endowments. . . . . 130

3. 3. Life Insurances . . . . . 133

3. 4. Endowments . . . . . 147

5. Life Annuities . . . . . 154

CHAPTER 4. PREMIUMS . . . . . 194

4. 1. Net Premiums . . . . . 194

4. 2. Gross Premiums . . . . . 215

VII CHAPTER 5. RESERVES . . . . . 223

5. 1. Net Premium Reserves . . . . . 223

5. 2. Mortality Profit. . . . . 272

5. 3. Modified Reserves . . . . . 286

ANSWERS TO ODD-NuMBERED PROBLEMS . . . . .

A new textbook offering a comprehensive introduction to models and techniques for the emerging field of actuarial Finance Drs. Boudreault and Renaud answer the need for a clear, application-oriented guide to the growing field of actuarial finance with this volume, which focuses on the mathematical models and techniques used in actuarial finance for the pricing and hedging of actuarial liabilities exposed to financial markets and other contingencies. With roots in modern financial mathematics, actuarial finance presents unique challenges due to the long-term nature of insurance liabilities, the presence of mortality or other contingencies and the structure and regulations of the insurance and pension markets. Motivated, designed and written for and by actuaries, this book puts actuarial applications at the forefront in addition to balancing mathematics and finance at an adequate level to actuarial undergraduates. While the classical theory of financial mathematics is discussed, the authors provide a thorough grounding in such crucial topics as recognizing embedded options in actuarial liabilities, adequately quantifying and pricing liabilities, and using derivatives and other assets to manage actuarial and financial risks. Actuarial applications are emphasized and illustrated with about 300 examples and 200 exercises. The book also comprises end-of-chapter point-form summaries to help the reader review the most important concepts. Additional topics and features include: Compares pricing in insurance and financial markets Discusses event-triggered derivatives such as weather, catastrophe and longevity derivatives and how they can be used for risk management; Introduces equity-linked insurance and annuities (EIAs, VAs), relates them to common derivatives and how to manage mortality for these products Introduces pricing and replication in incomplete markets and analyze the impact of market incompleteness on insurance and risk management; Presents immunization techniques alongside Greeks-based hedging; Covers in detail how to delta-gamma/rho/vega hedge a liability and how to rebalance periodically a hedging portfolio. This text will prove itself a firm foundation for undergraduate courses in financial mathematics or economics, actuarial mathematics or derivative markets. It is also highly applicable to current and future actuaries preparing for the exams or actuary professionals looking for a valuable addition to their reference shelf. As of 2019, the book covers significant parts of the Society of Actuaries' Exams FM, IFM and QFI Core, and the Casualty Actuarial Society's Exams 2 and 3F. It is assumed the reader has basic skills in calculus (differentiation and integration of functions), probability (at

the level of the Society of Actuaries' Exam P), interest theory (time value of money) and, ideally, a basic understanding of elementary stochastic processes such as random walks.

This book provides a thorough understanding of the fundamental concepts of financial mathematics essential for the evaluation of any financial product and instrument. Mastering concepts of present and future values of streams of cash flows under different interest rate environments is core for actuaries and financial economists. This book covers the body of knowledge required by the Society of Actuaries (SOA) for its Financial Mathematics (FM) Exam. The third edition includes major changes such as an addition of an 'R Laboratory' section in each chapter, except for Chapter 9. These sections provide R codes to do various computations, which will facilitate students to apply conceptual knowledge. Additionally, key definitions have been revised and the theme structure has been altered. Students studying undergraduate courses on financial mathematics for actuaries will find this book useful. This book offers numerous examples and exercises, some of which are adapted from previous SOA FM Exams. It is also useful for students preparing for the actuarial professional exams through self-study.

This must-have manual provides detailed solutions to all of the 200+ exercises in Dickson, Hardy and Waters' Actuarial Mathematics for Life Contingent Risks, Second Edition. This groundbreaking text on the modern mathematics of life insurance is required reading for the Society of Actuaries' Exam MLC and also provides a solid preparation for the life contingencies material of the UK actuarial profession's exam CT5. Beyond the professional examinations, the textbook and solutions manual offer readers the opportunity to develop insight and understanding, and also offer practical advice for solving problems using straightforward, intuitive numerical methods. Companion spreadsheets illustrating these techniques are available for free download.

Describes the application of actuarial principles and techniques to public social insurance pension schemes. Aims to establish a link between public social security and occupational pension scheme methods. Part one discusses actuarial theory. Part two deals with two techniques: the projection technique, and the present value technique. There is also a brief description of actuarial mathematics.

Financial Mathematics For Actuaries (Second Edition) World Scientific Publishing Company

Taking classic stories from Asia and the West, Pop! Lit for Kids reimagines them into easy-to-read stories that provide the perfect introduction to classic tales. The most well-loved stories from around the world have been adapted into a form that will excite and entertain children everywhere. Readers can embark on new adventures with famous beloved storybook characters. In addition, the books come to life with augmented reality features, giving readers an enhanced experience that they'll never forget!

The 800 years of scientific breakthroughs that will help salvage your retirement plans

Physics, Chemistry, Astronomy, Biology; every field has its intellectual giants who made breakthrough discoveries that changed the course of history. What about the topic of retirement planning? Is it a science? Or is retirement income planning just a collection of rules-of-thumb, financial products and sales pitches? In *The 7 Most Important Equations for Your Retirement... And the Stories Behind Them* Moshe Milevsky argues that twenty-first century retirement income planning is indeed a science and has its foundations in the work of great sages who made conceptual and controversial breakthroughs over the last eight centuries. In the book Milevsky highlights the work of seven scholars—summarized by seven equations—who shaped all modern retirement calculations. He tells the stories of Leonardo Fibonacci the Italian businessman; Benjamin Gompertz the gentleman actuary; Edmund Halley the astronomer; Irving Fisher the stock jock; Paul Samuelson the economic guru; Solomon Heubner the insurance and marketing visionary, and Andrey Kolmogorov the Russian mathematical genius—all giants in their respective fields who collectively laid the foundations for modern retirement income planning. With baby boomers starting to hit retirement age, planning for retirement income has become a hot topic across the country. Author Moshe Milevsky is an internationally-respected financial expert with the knowledge you need to assess whether you are ready to retire or not. Presents an entertaining, informative narrative approach to financial planning. Understanding the ideas behind these seven foundation equations—which Moshe Milevsky explains in a manner that everyone can appreciate—will help baby boomers better prepare for retirement. This is a book unlike anything you have ever read on retirement planning. Think Suze Orman meets Stephen Hawking. If you ever wondered what the point of all that high school mathematics was, Moshe Milevsky's answer is: So that you can figure out how to retire... while you can still enjoy your money.

*Financial Mathematics for Actuarial Science: The Theory of Interest* is concerned with the measurement of interest and the various ways interest affects what is often called the time value of money (TVM). Interest is most simply defined as the compensation that a borrower pays to a lender for the use of capital. The goal of this book is to provide the mathematical understandings of interest and the time value of money needed to succeed on the actuarial examination covering interest theory. Key Features: Helps prepare students for the SOA Financial Mathematics Exam. Provides mathematical understanding of interest and the time value of money needed to succeed in the actuarial examination covering interest theory. Contains many worked examples, exercises and solutions for practice. Provides training in the use of calculators for solving problems. A complete solutions manual is available to faculty adopters online. The 1922 volume was, in turn, created as the replacement for the Institute of Actuaries Textbook, Part Three.

1. The Measurement of Interest ;
2. Solution of Problems in Interest ;
3. Elementary Annuities ;
4. More General Annuities ;
5. Yield Rates ;
6. Amortization Schedules and Sinking Funds ;
7. Bond and Other Securities ;
8. Practical Applications ;
9. More Advanced Financial Analysis ;
10. A Stochastic Approach to Interest ;

APPENDIXES I. Table of compound interest functions ; II. Table numbering the days of the year ; III. Basic mathematical review ; IV. Statistical background ; V. An introduction to finite differences ; VI. Iteration methods ; VII. Further analysis of varying annuities ; VIII. A general formula for amortization with step-rate amounts of principle ; Bibliography ;



Answers to the exercises ; Index.

This self-contained module for independent study covers the subjects most often needed by non-mathematics graduates, such as fundamental calculus, linear algebra, probability, and basic numerical methods. The easily-understandable text of Introduction to Actuarial and Mathematical Methods features examples, motivations, and lots of practice from a large number of end-of-chapter questions. For readers with diverse backgrounds entering programs of the Institute and Faculty of Actuaries, the Society of Actuaries, and the CFA Institute, Introduction to Actuarial and Mathematical Methods can provide a consistency of mathematical knowledge from the outset. Presents a self-study mathematics refresher course for the first two years of an actuarial program Features examples, motivations, and practice problems from a large number of end-of-chapter questions designed to promote independent thinking and the application of mathematical ideas Practitioner friendly rather than academic Ideal for self-study and as a reference source for readers with diverse backgrounds entering programs of the Institute and Faculty of Actuaries, the Society of Actuaries, and the CFA Institute

An Introduction to the Mathematics of Finance: A Deterministic Approach, 2e, offers a highly illustrated introduction to mathematical finance, with a special emphasis on interest rates. This revision of the McCutcheon-Scott classic follows the core subjects covered by the first professional exam required of UK actuaries, the CT1 exam. It realigns the table of contents with the CT1 exam and includes sample questions from past exams of both The Actuarial Profession and the CFA Institute. With a wealth of solved problems and interesting applications, An Introduction to the Mathematics of Finance stands alone in its ability to address the needs of its primary target audience, the actuarial student. Closely follows the syllabus for the CT1 exam of The Institute and Faculty of Actuaries Features new content and more examples Online supplements available: <http://booksite.elsevier.com/9780080982403/> Includes past exam questions from The Institute and Faculty of Actuaries and the CFA Institute

This second edition expands the first chapters, which focus on the approach to risk management issues discussed in the first edition, to offer readers a better understanding of the risk management process and the relevant quantitative phases. In the following chapters the book examines life insurance, non-life insurance and pension plans, presenting the technical and financial aspects of risk transfers and insurance without the use of complex mathematical tools. The book is written in a comprehensible style making it easily accessible to advanced undergraduate and graduate students in Economics, Business and Finance, as well as undergraduate students in Mathematics who intend starting on an actuarial qualification path. With the systematic inclusion of practical topics, professionals will find this text useful when working in insurance and pension related areas, where investments, risk analysis and financial reporting play a major role.

Risk Neutral Pricing and Financial Mathematics: A Primer provides a foundation to financial mathematics for those whose undergraduate quantitative preparation does not extend beyond calculus, statistics, and linear math. It covers a broad range of foundation topics related to financial modeling, including probability, discrete and continuous time and space valuation, stochastic processes, equivalent martingales, option pricing, and term structure models, along with related valuation and hedging

techniques. The joint effort of two authors with a combined 70 years of academic and practitioner experience, *Risk Neutral Pricing and Financial Mathematics* takes a reader from learning the basics of beginning probability, with a refresher on differential calculus, all the way to Doob-Meyer, Ito, Girsanov, and SDEs. It can also serve as a useful resource for actuaries preparing for Exams FM and MFE (Society of Actuaries) and Exams 2 and 3F (Casualty Actuarial Society). Includes more subjects than other books, including probability, discrete and continuous time and space valuation, stochastic processes, equivalent martingales, option pricing, term structure models, valuation, and hedging techniques Emphasizes introductory financial engineering, financial modeling, and financial mathematics Suited for corporate training programs and professional association certification programs

Understand Up-to-Date Statistical Techniques for Financial and Actuarial

Applications Since the first edition was published, statistical techniques, such as reliability measurement, simulation, regression, and Markov chain modeling, have become more prominent in the financial and actuarial industries. Consequently, practitioners and students must ac

This book explains what actuaries are, what they do, and where they do it. It describes the ideas, techniques, and skills involved in the day-to-day work of actuaries. This second edition has been updated to reflect the rise of social networking and the internet, the progress toward a global knowledge-based economy, and the global expansion of the actuarial field that has occurred since the first edition. --from publisher description

*Financial Mathematics for Actuaries* is a textbook for students in actuarial science, quantitative finance, financial engineering and quantitative risk management and is designed for a one-semester undergraduate course. Covering the theories of interest rates, with applications to the evaluation of cash flows, the pricing of fixed income securities and the management of bonds, this textbook also contains numerous examples and exercises and extensive coverage of various Excel functions for financial calculation. Discussions are linked to real financial market data, such as historical term structure, and traded financial securities. The topics discussed in this book are essential for actuarial science students. They are also useful for students in financial markets, investments and quantitative finance. Students preparing for examinations in financial mathematics with various professional actuarial bodies will also find this book useful for self-study. In this second edition, the recent additions in the learning objectives of the Society of Actuaries Exam FM have been covered.

This class-tested undergraduate textbook covers the entire syllabus for Exam C of the Society of Actuaries (SOA).

*Mathematical Interest Theory* provides an introduction to how investments grow over time. This is done in a mathematically precise manner. The emphasis is on practical applications that give the reader a concrete understanding of why the various relationships should be true. Among the modern financial topics introduced are: arbitrage, options, futures, and swaps. *Mathematical Interest Theory* is written for anyone who has a strong high-school algebra background and is interested in being an informed borrower or investor. The book is suitable for a mid-level or upper-level undergraduate course or a beginning graduate course. The content of the book, along with an understanding of probability, will provide a solid foundation for readers

embarking on actuarial careers. The text has been suggested by the Society of Actuaries for people preparing for the Financial Mathematics exam. To that end, *Mathematical Interest Theory* includes more than 260 carefully worked examples. There are over 475 problems, and numerical answers are included in an appendix. A companion student solution manual has detailed solutions to the odd-numbered problems. Most of the examples involve computation, and detailed instruction is provided on how to use the Texas Instruments BA II Plus and BA II Plus Professional calculators to efficiently solve the problems. This Third Edition updates the previous edition to cover the material in the SOA study notes FM-24-17, FM-25-17, and FM-26-17.

The proliferation of financial derivatives over the past decades, options in particular, has underscored the increasing importance of derivative pricing literacy among students, researchers, and practitioners. *Derivative Pricing: A Problem-Based Primer* demystifies the essential derivative pricing theory by adopting a mathematically rigorous yet widely accessible pedagogical approach that will appeal to a wide variety of audience. Abandoning the traditional "black-box" approach or theorists' "pedantic" approach, this textbook provides readers with a solid understanding of the fundamental mechanism of derivative pricing methodologies and their underlying theory through a diversity of illustrative examples. The abundance of exercises and problems makes the book well-suited as a text for advanced undergraduates, beginning graduates as well as a reference for professionals and researchers who need a thorough understanding of not only "how," but also "why" derivative pricing works. It is especially ideal for students who need to prepare for the derivatives portion of the Society of Actuaries Investment and Financial Markets Exam. Features Lucid explanations of the theory and assumptions behind various derivative pricing models. Emphasis on intuitions, mnemonics as well as common fallacies. Interspersed with illustrative examples and end-of-chapter problems that aid a deep understanding of concepts in derivative pricing. Mathematical derivations, while not eschewed, are made maximally accessible. A solutions manual is available for qualified instructors. The Author Ambrose Lo is currently Assistant Professor of Actuarial Science at the Department of Statistics and Actuarial Science at the University of Iowa. He received his Ph.D. in Actuarial Science from the University of Hong Kong in 2014, with dependence structures, risk measures, and optimal reinsurance being his research interests. He is a Fellow of the Society of Actuaries (FSA) and a Chartered Enterprise Risk Analyst (CERA). His research papers have been published in top-tier actuarial journals, such as *ASTIN Bulletin: The Journal of the International Actuarial Association*, *Insurance: Mathematics and Economics*, and *Scandinavian Actuarial Journal*.

This volume aims to collect new ideas presented in the form of 4 page papers dedicated to mathematical and statistical methods in actuarial sciences and finance. The cooperation between mathematicians and statisticians working in insurance and finance is a very fruitful field and provides interesting scientific products in theoretical models and practical applications, as well as in scientific discussion of problems of national and international interest. This work reflects the results discussed at the biennial conference on *Mathematical and Statistical Methods for Actuarial Sciences and Finance (MAF)*, born at the University of Salerno in 2004.

Versatile for Several Interrelated Courses at the Undergraduate and Graduate Levels *Financial Mathematics: A Comprehensive Treatment* provides a unified, self-contained account of the main theory and application of methods behind modern-day financial mathematics. Tested and refined through years of the authors' teaching experiences, the book encompasses a breadth of topics, from introductory to more advanced ones. Accessible to undergraduate students in mathematics, finance, actuarial science, economics, and related quantitative areas, much of

the text covers essential material for core curriculum courses on financial mathematics. Some of the more advanced topics, such as formal derivative pricing theory, stochastic calculus, Monte Carlo simulation, and numerical methods, can be used in courses at the graduate level. Researchers and practitioners in quantitative finance will also benefit from the combination of analytical and numerical methods for solving various derivative pricing problems. With an abundance of examples, problems, and fully worked out solutions, the text introduces the financial theory and relevant mathematical methods in a mathematically rigorous yet engaging way. Unlike similar texts in the field, this one presents multiple problem-solving approaches, linking related comprehensive techniques for pricing different types of financial derivatives. The book provides complete coverage of both discrete- and continuous-time financial models that form the cornerstones of financial derivative pricing theory. It also presents a self-contained introduction to stochastic calculus and martingale theory, which are key fundamental elements in quantitative finance.

Marky's cousin Martial Polo invites him to Beijing, after hearing of his first trip overseas to Tokyo. Martial Polo is a Wushu enthusiast who dreams of being a renowned martial arts expert like Great-grandmother MuayThai Polo. Martial turns Marky's holiday into an exhausting strength-building, endurance-training workout. Can Marky endure Martial's bossy nature and find a way to enjoy sightseeing in Beijing at a slower pace?

This book teaches multiple regression and time series and how to use these to analyze real data in risk management and finance.

This book provides a comprehensive introduction to actuarial mathematics, covering both deterministic and stochastic models of life contingencies, as well as more advanced topics such as risk theory, credibility theory and multi-state models. This new edition includes additional material on credibility theory, continuous time multi-state models, more complex types of contingent insurances, flexible contracts such as universal life, the risk measures VaR and TVaR. Key Features: Covers much of the syllabus material on the modeling examinations of the Society of Actuaries, Canadian Institute of Actuaries and the Casualty Actuarial Society. (SOA-CIA exams MLC and C, CSA exams 3L and 4.) Extensively revised and updated with new material. Orders the topics specifically to facilitate learning. Provides a streamlined approach to actuarial notation. Employs modern computational methods. Contains a variety of exercises, both computational and theoretical, together with answers, enabling use for self-study. An ideal text for students planning for a professional career as actuaries, providing a solid preparation for the modeling examinations of the major North American actuarial associations. Furthermore, this book is highly suitable reference for those wanting a sound introduction to the subject, and for those working in insurance, annuities and pensions.

Actuarial Models: The Mathematics of Insurance, Second Edition thoroughly covers the basic models of insurance processes. It also presents the mathematical frameworks and methods used in actuarial modeling. This second edition provides an even smoother, more robust account of the main ideas and models, preparing students to take exams of the Society

This edition of the private and scientific correspondence of Sir Rudolf Peierls gives a unique insight into the life and work of one of the greatest theoretical physicists of the 20th century. Rudolf Peierls' scientific work contributed to the early developments in quantum mechanics, and he is well known and much appreciated for his contributions to various disciplines, including solid state physics, nuclear physics, and particle physics. As an enthusiastic and devoted teacher, he passed on his knowledge and understanding and inspired the work of collaborators and students alike. As an effective administrator he was responsible, almost single-handedly, for the establishment of an outstanding successful centre of theoretical physics in Birmingham, and later contributed much to theoretical physics in Oxford. A meticulous collector of



correspondence, Sir Rudolf left a fascinating collection of letters, in some cases spanning more than seven decades. This collection includes correspondence with his parents, his wife, the Russian-born physicist Genia Kannegieser, life-long friends such as Hans Bethe, and many great physicists, including Wolfgang Pauli, Niels Bohr, Werner Heisenberg, Lev Landau, and George Placzek, to name but a few. This first volume, which covers the years 1922 to 1945, contains much of the early family correspondence, letters exchanged between Rudolf and Genia Peierls before and after their marriage in 1931, correspondence relating to early developments in quantum physics, and interesting material relating to the development of nuclear weapons. The extensive apparatus provides an invaluable background which allows the reader to put the presented documents into their multi-faceted social, political and scientific context. In this Special Edition comic book in the bestselling World of Science series, A\* STAR scientist Germaine Yong imagines the human digestive system as an exciting journey for a young bacteria named Guss. Learn about the ins and outs of our digestive system and how to stay healthy, while having fun with Guss. And, augmented reality features will bring Science learning to life and engage readers of all ages!

This book provides a thorough understanding of the fundamental concepts of financial mathematics essential for the evaluation of any financial product and instrument. Mastering concepts of present and future values of streams of cash flows under different interest rate environments is core for actuaries and financial economists. This book covers the body of knowledge required by the Society of Actuaries (SOA) for its Financial Mathematics (FM) Exam. The third edition includes major changes such as an addition of an R Laboratory section in each chapter, except for Chapter 9. These sections provide R codes to do various computations, which will facilitate students to apply conceptual knowledge. Additionally, key definitions have been revised and the theme structure has been altered. Students studying undergraduate courses on financial mathematics for actuaries will find this book useful. This book offers numerous examples and exercises, some of which are adapted from previous SOA FM Exams. It is also useful for students preparing for the actuarial professional exams through self-study.

This text aims to help readers become “literate” in the vocabulary of finance, insurance, and pensions and be able to utilize the appropriate mathematics for professional and personal use. This book covers a wide range of topics not found in other texts, including complex annuities, complex perpetuities, geometrically varying annuities, and bond duration and volatility. This book is a helpful reference to all professionals in the fields of accounting, finance and financial services, management, marketing services, computer information systems, and economics. It is also ideal for anyone who wants a self-study for personal finances.

A Hands-On Approach to Understanding and Using Actuarial Models Computational Actuarial Science with R provides an introduction to the computational aspects of actuarial science. Using simple R code, the book helps you understand the algorithms involved in actuarial computations. It also covers more advanced topics, such as parallel computing and C/C++ embedded codes. After an introduction to the R language, the book is divided into four parts. The first one addresses methodology and statistical modeling issues. The second part discusses the computational facets of life insurance, including life contingencies calculations and prospective life tables. Focusing

on finance from an actuarial perspective, the next part presents techniques for modeling stock prices, nonlinear time series, yield curves, interest rates, and portfolio optimization. The last part explains how to use R to deal with computational issues of nonlife insurance. Taking a do-it-yourself approach to understanding algorithms, this book demystifies the computational aspects of actuarial science. It shows that even complex computations can usually be done without too much trouble. Datasets used in the text are available in an R package (CASdatasets).

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