

Exercises In Functional Analysis 1st Edition

This book covers functional analysis and its applications to continuum mechanics.; The mathematical material is treated in a non-abstract manner and is fully illuminated by the underlying mechanical ideas.; The presentation is concise but complete, and is intended for specialists in continuum mechanics who wish to understand the mathematical underpinnings of the discipline.; Graduate students and researchers in mathematics, physics, and engineering will find this book useful.; Exercises and examples are included throughout with detailed solutions provided in the appendix. This textbook is an introduction to functional analysis suited to final year undergraduates or beginning graduates. Its various applications of Hilbert spaces, including least squares approximation, inverse problems, and Tikhonov regularization, should appeal not only to mathematicians interested in applications, but also to researchers in related fields. Functional Analysis adopts a self-contained approach to Banach spaces and operator theory that covers the main topics, based upon the classical sequence and function spaces and their operators. It assumes only a minimum of knowledge in elementary linear algebra and real analysis; the latter is redone in the light of metric spaces. It contains more than a thousand worked examples and exercises, which make up the main body of the book.

This book is meant as a text for a first-year graduate course in analysis. In a sense, it covers the same topics as elementary calculus but treats them in a manner suitable for people who will be using it in further mathematical investigations. The organization avoids long chains of logical interdependence, so that chapters are mostly independent. This allows a course to omit material from some chapters without compromising the exposition of material from later chapters.

to the English Translation This is a concise guide to basic sections of modern functional analysis. Included are such topics as the principles of Banach and Hilbert spaces, the theory of multinormed and uniform spaces, the Riesz-Dunford holomorphic functional calculus, the Fredholm index theory, convex analysis and duality theory for locally convex spaces. With standard provisos the presentation is self-contained, exposing about a hundred famous "named" theorems furnished with complete proofs and culminating in the Gelfand-Naimark-Segal construction for C^* -algebras. The first Russian edition was printed by the Siberian Division of "Nauka" Publishers in 1983. Since then the monograph has served as the standard textbook on functional analysis at the University of Novosibirsk. This volume is translated from the second Russian edition printed by the Sobolev Institute of Mathematics of the Siberian Division of the Russian Academy of Sciences in 1995. It incorporates new sections on Radon measures, the Schwartz spaces of distributions, and a supplementary list of theoretical exercises and problems. This edition was typeset using AMS- \LaTeX , the American Mathematical Society's \LaTeX system. To clear my conscience completely, I also confess that $:=$ stands for the definor, the assignment operator, signifies the end of the proof. This introduction to the ideas and methods of linear functional analysis shows how familiar and useful concepts from finite-dimensional linear algebra can be extended or generalized to infinite-dimensional spaces. Aimed at advanced undergraduates in mathematics and physics, the book assumes a standard background of linear algebra, real analysis (including the theory of metric spaces), and Lebesgue integration,

although an introductory chapter summarizes the requisite material. A highlight of the second edition is a new chapter on the Hahn-Banach theorem and its applications to the theory of duality.

The material presented in this book is suited for a first course in Functional Analysis which can be followed by Masters students. While covering all the standard material expected of such a course, efforts have been made to illustrate the use of various theorems via examples taken from differential equations and the calculus of variations, either through brief sections or through exercises. In fact, this book will be particularly useful for students who would like to pursue a research career in the applications of mathematics. The book includes a chapter on weak and weak topologies and their applications to the notions of reflexivity, separability and uniform convexity. The chapter on the Lebesgue spaces also presents the theory of one of the simplest classes of Sobolev spaces. The book includes a chapter on compact operators and the spectral theory for compact self-adjoint operators on a Hilbert space. Each chapter has large collection of exercises at the end. These illustrate the results of the text, show the optimality of the hypotheses of various theorems via examples or counterexamples, or develop simple versions of theories not elaborated upon in the text.

Written by an expert on the topic and experienced lecturer, this textbook provides an elegant, self-contained introduction to functional analysis, including several advanced topics and applications to harmonic analysis. Starting from basic topics before proceeding to more advanced material, the book covers measure and integration theory, classical Banach and Hilbert space theory, spectral theory for bounded operators, fixed point theory, Schauder bases, the Riesz-Thorin interpolation theorem for operators, as well as topics in duality and convexity theory. Aimed at advanced undergraduate and graduate students, this book is suitable for both introductory and more advanced courses in functional analysis. Including over 1500 exercises of varying difficulty and various motivational and historical remarks, the book can be used for self-study and alongside lecture courses.

The book is based on courses taught by the author at Moscow State University. Compared to many other books on the subject, it is unique in that the exposition is based on extensive use of the language and elementary constructions of category theory. Among topics featured in the book are the theory of Banach and Hilbert tensor products, the theory of distributions and weak topologies, and Borel operator calculus. The book contains many examples illustrating the general theory presented, as well as multiple exercises that help the reader to learn the subject. It can be used as a textbook on selected topics of functional analysis and operator theory. Prerequisites include linear algebra, elements of real analysis, and elements of the theory of metric spaces. Introductory text covers basic structures of mathematical analysis (linear spaces, metric spaces, normed linear spaces, etc.), differential equations, orthogonal expansions, Fourier transforms, and more. Includes problems with hints and answers. Bibliography. 1974 edition.

This book of exercises in Functional Analysis contains almost 450 exercises (all with complete solutions), providing supplementary examples, counter-examples and applications for the basic notions usually presented in an introductory course in Functional Analysis. It contains three parts. The first one contains exercises on the general properties for sets in normed spaces, linear bounded operators on normed

spaces, reflexivity, compactness in normed spaces, and on the basic principles in Functional Analysis: the Hahn-Banach theorem, the Uniform Boundedness Principle, the Open Mapping and the Closed Graph theorems. The second one contains exercises on the general theory of Hilbert spaces, the Riesz representation theorem, orthogonality in Hilbert spaces, the projection theorem and linear bounded operators on Hilbert spaces. The third one deals with linear topological spaces, and includes a large number of exercises on the weak topologies.

A powerful introduction to one of the most active areas of theoretical and applied mathematics. This distinctive introduction to one of the most far-reaching and beautiful areas of mathematics focuses on Banach spaces as the milieu in which most of the fundamental concepts are presented. While occasionally using the more general topological vector space and locally convex space setting, it emphasizes the development of the reader's mathematical maturity and the ability to both understand and "do" mathematics. In so doing, Functional Analysis provides a strong springboard for further exploration on the wide range of topics the book presents, including: * Weak topologies and applications * Operators on Banach spaces * Bases in Banach spaces * Sequences, series, and geometry in Banach spaces. Stressing the general techniques underlying the proofs, Functional Analysis also features many exercises for immediate clarification of points under discussion. This thoughtful, well-organized synthesis of the work of those mathematicians who created the discipline of functional analysis as we know it today also provides a rich source of research topics and reference material. Functional analysis arose in the early twentieth century and gradually, conquering one stronghold after another, became a nearly universal mathematical doctrine, not merely a new area of mathematics, but a new mathematical world view. Its appearance was the inevitable consequence of the evolution of all of nineteenth-century mathematics, in particular classical analysis and mathematical physics. Its original basis was formed by Cantor's theory of sets and linear algebra. Its existence answered the question of how to state general principles of a broadly interpreted analysis in a way suitable for the most diverse situations. A.M. Vershik ([45], p. 438). This text evolved from the content of a one semester introductory course in functional analysis that I have taught a number of times since 1996 at the University of Virginia. My students have included first and second year graduate students preparing for thesis work in analysis, algebra, or topology, graduate students in various departments in the School of Engineering and Applied Science, and several undergraduate mathematics or physics majors. After a first draft of the manuscript was completed, it was also used for an independent reading course for several undergraduates preparing for graduate school.

A concise introduction to the major concepts of functional analysis. Requiring only a preliminary knowledge of elementary linear algebra and real analysis, *A First Course in Functional Analysis* provides an introduction to the basic principles and practical applications of functional analysis. Key concepts are illustrated in a straightforward manner, which facilitates a complete and fundamental understanding of the topic. This book is based on the author's own class-tested material and uses clear language to explain the major concepts of functional analysis, including Banach spaces, Hilbert spaces, topological vector spaces, as well as bounded linear functionals and operators. As opposed to simply presenting the proofs, the author outlines the logic behind the steps, demonstrates the development of arguments, and discusses how the concepts

are connected to one another. Each chapter concludes with exercises ranging in difficulty, giving readers the opportunity to reinforce their comprehension of the discussed methods. An appendix provides a thorough introduction to measure and integration theory, and additional appendices address the background material on topics such as Zorn's lemma, the Stone-Weierstrass theorem, Tychonoff's theorem on product spaces, and the upper and lower limit points of sequences. References to various applications of functional analysis are also included throughout the book. A First Course in Functional Analysis is an ideal text for upper-undergraduate and graduate-level courses in pure and applied mathematics, statistics, and engineering. It also serves as a valuable reference for practitioners across various disciplines, including the physical sciences, economics, and finance, who would like to expand their knowledge of functional analysis.

This book offers a brief, practically complete, and relatively simple introduction to functional analysis. It also illustrates the application of functional analytic methods to the science of continuum mechanics. Abstract but powerful mathematical notions are tightly interwoven with physical ideas in the treatment of nontrivial boundary value problems for mechanical objects. This second edition includes more extended coverage of the classical and abstract portions of functional analysis. Taken together, the first three chapters now constitute a regular text on applied functional analysis. This potential use of the book is supported by a significantly extended set of exercises with hints and solutions. A new appendix, providing a convenient listing of essential inequalities and imbedding results, has been added. The book should appeal to graduate students and researchers in physics, engineering, and applied mathematics. Reviews of first edition: "This book covers functional analysis and its applications to continuum mechanics. The presentation is concise but complete, and is intended for readers in continuum mechanics who wish to understand the mathematical underpinnings of the discipline. ... Detailed solutions of the exercises are provided in an appendix." (L'Enseignement Mathématique, Vol. 49 (1-2), 2003) "The reader comes away with a profound appreciation both of the physics and its importance, and of the beauty of the functional analytic method, which, in skillful hands, has the power to dissolve and clarify these difficult problems as peroxide does clotted blood. Numerous exercises ... test the reader's comprehension at every stage. Summing Up: Recommended." (F. E. J. Linton, Choice, September, 2003)

This book gives an introduction to Linear Functional Analysis, which is a synthesis of algebra, topology, and analysis. In addition to the basic theory it explains operator theory, distributions, Sobolev spaces, and many other things. The text is self-contained and includes all proofs, as well as many exercises, most of them with solutions. Moreover, there are a number of appendices, for example on Lebesgue integration theory. A complete introduction to the subject, Linear Functional Analysis will be particularly useful to readers who want to quickly get to the key statements and who are interested in applications to differential equations.

This text offers a survey of the main ideas, concepts, and methods that constitute nonlinear functional analysis. It features extensive commentary, many examples, and interesting, challenging exercises. 1985 edition.

This classic text is written for graduate courses in functional analysis. This text is used in modern investigations in analysis and applied mathematics. This new edition includes up-to-

date presentations of topics as well as more examples and exercises. New topics include Kakutani's fixed point theorem, Lomonosov's invariant subspace theorem, and an ergodic theorem. This text is part of the Walter Rudin Student Series in Advanced Mathematics. Accessible text covering core functional analysis topics in Hilbert and Banach spaces, with detailed proofs and 200 fully-worked exercises.

Provides avenues for applying functional analysis to the practical study of natural sciences as well as mathematics. Contains worked problems on Hilbert space theory and on Banach spaces and emphasizes concepts, principles, methods and major applications of functional analysis.

Functional analysis is a broad mathematical area with strong connections to many domains within mathematics and physics. This book, based on a first-year graduate course taught by Robert J. Zimmer at the University of Chicago, is a complete, concise presentation of fundamental ideas and theorems of functional analysis. It introduces essential notions and results from many areas of mathematics to which functional analysis makes important contributions, and it demonstrates the unity of perspective and technique made possible by the functional analytic approach. Zimmer provides an introductory chapter summarizing measure theory and the elementary theory of Banach and Hilbert spaces, followed by a discussion of various examples of topological vector spaces, seminorms defining them, and natural classes of linear operators. He then presents basic results for a wide range of topics: convexity and fixed point theorems, compact operators, compact groups and their representations, spectral theory of bounded operators, ergodic theory, commutative C^* -algebras, Fourier transforms, Sobolev embedding theorems, distributions, and elliptic differential operators. In treating all of these topics, Zimmer's emphasis is not on the development of all related machinery or on encyclopedic coverage but rather on the direct, complete presentation of central theorems and the structural framework and examples needed to understand them. Sets of exercises are included at the end of each chapter. For graduate students and researchers in mathematics who have mastered elementary analysis, this book is an entrée and reference to the full range of theory and applications in which functional analysis plays a part. For physics students and researchers interested in these topics, the lectures supply a thorough mathematical grounding. This text presents selected areas of functional analysis that can facilitate an understanding of ideas in probability and stochastic processes. Topics covered include basic Hilbert and Banach spaces, weak topologies and Banach algebras, and the theory of semigroups of bounded linear operators.

This book is an introductory text in functional analysis. Unlike many modern treatments, it begins with the particular and works its way to the more general. From the reviews: "This book is an excellent text for a first graduate course in functional analysis....Many interesting and important applications are included....It includes an abundance of exercises, and is written in the engaging and lucid style which we have come to expect from the author."

--MATHEMATICAL REVIEWS

This book contains almost 450 exercises, all with complete solutions; it provides supplementary examples, counter-examples, and applications for the basic notions usually presented in an introductory course in Functional Analysis. Three comprehensive sections cover the broad topic of functional analysis. A large number of exercises on the weak topologies is included.

Exercises in Functional Analysis Springer Science & Business Media

This textbook is a completely revised, updated, and expanded English edition of the important *Analyse fonctionnelle* (1983). In addition, it contains a wealth of problems and exercises (with solutions) to guide the reader. Uniquely, this book presents in a coherent, concise and unified way the main results from functional

analysis together with the main results from the theory of partial differential equations (PDEs). Although there are many books on functional analysis and many on PDEs, this is the first to cover both of these closely connected topics. Since the French book was first published, it has been translated into Spanish, Italian, Japanese, Korean, Romanian, Greek and Chinese. The English edition makes a welcome addition to this list.

"Functional analysis studies the algebraic, geometric, and topological structures of spaces and operators that underlie many classical problems. Individual functions satisfying specific equations are replaced by classes of functions and transforms that are determined by the particular problems at hand. This book presents the basic facts of linear functional analysis as related to fundamental aspects of mathematical analysis and their applications. The exposition avoids unnecessary terminology and generality and focuses on showing how the knowledge of these structures clarifies what is essential in analytic problems. The material in the first part of the book can be used for an introductory course on functional analysis, with an emphasis on the role of duality. The second part introduces distributions and Sobolev spaces and their applications. Convolution and the Fourier transform are shown to be useful tools for the study of partial differential equations. Fundamental solutions and Green's functions are considered and the theory is illustrated with several applications. In the last chapters, the Gelfand transform for Banach algebras is used to present the spectral theory of bounded and unbounded operators, which is then used in an introduction to the basic axioms of quantum mechanics. The presentation is intended to be accessible to readers whose backgrounds include basic linear algebra, integration theory, and general topology. Almost 240 exercises will help the reader in better understanding the concepts employed."--Publisher's description.

Even the simplest mathematical abstraction of the phenomena of reality the real line-can be regarded from different points of view by different mathematical disciplines. For example, the algebraic approach to the study of the real line involves describing its properties as a set to whose elements we can apply "operations," and obtaining an algebraic model of it on the basis of these properties, without regard for the topological properties. On the other hand, we can focus on the topology of the real line and construct a formal model of it by singling out its "continuity" as a basis for the model. Analysis regards the line, and the functions on it, in the unity of the whole system of their algebraic and topological properties, with the fundamental deductions about them obtained by using the interplay between the algebraic and topological structures. The same picture is observed at higher stages of abstraction. Algebra studies linear spaces, groups, rings, modules, and so on. Topology studies structures of a different kind on arbitrary sets, structures that give mathematical meaning to the concepts of a limit, continuity, a neighborhood, and so on. Functional analysis takes up topological linear spaces, topological groups, normed rings, modules of

representations of topological groups in topological linear spaces, and so on. Thus, the basic object of study in functional analysis consists of objects equipped with compatible algebraic and topological structures.

This book provides a unique path for graduate or advanced undergraduate students to begin studying the rich subject of functional analysis with fewer prerequisites than is normally required. The text begins with a self-contained and highly efficient introduction to topology and measure theory, which focuses on the essential notions required for the study of functional analysis, and which are often buried within full-length overviews of the subjects. This is particularly useful for those in applied mathematics, engineering, or physics who need to have a firm grasp of functional analysis, but not necessarily some of the more abstruse aspects of topology and measure theory normally encountered. The reader is assumed to only have knowledge of basic real analysis, complex analysis, and algebra. The latter part of the text provides an outstanding treatment of Banach space theory and operator theory, covering topics not usually found together in other books on functional analysis. Written in a clear, concise manner, and equipped with a rich array of interesting and important exercises and examples, this book can be read for an independent study, used as a text for a two-semester course, or as a self-contained reference for the researcher.

Based on a graduate course by the celebrated analyst Nigel Kalton, this well-balanced introduction to functional analysis makes clear not only how, but why, the field developed. All major topics belonging to a first course in functional analysis are covered. However, unlike traditional introductions to the subject, Banach spaces are emphasized over Hilbert spaces, and many details are presented in a novel manner, such as the proof of the Hahn–Banach theorem based on an inf-convolution technique, the proof of Schauder's theorem, and the proof of the Milman–Pettis theorem. With the inclusion of many illustrative examples and exercises, *An Introductory Course in Functional Analysis* equips the reader to apply the theory and to master its subtleties. It is therefore well-suited as a textbook for a one- or two-semester introductory course in functional analysis or as a companion for independent study.

This textbook provides a careful treatment of functional analysis and some of its applications in analysis, number theory, and ergodic theory. In addition to discussing core material in functional analysis, the authors cover more recent and advanced topics, including Weyl's law for eigenfunctions of the Laplace operator, amenability and property (T), the measurable functional calculus, spectral theory for unbounded operators, and an account of Tao's approach to the prime number theorem using Banach algebras. The book further contains numerous examples and exercises, making it suitable for both lecture courses and self-study. *Functional Analysis, Spectral Theory, and Applications* is aimed at postgraduate and advanced undergraduate students with some background in analysis and algebra, but will also appeal to everyone with an interest in seeing how functional analysis can be applied to other parts of mathematics.

In an elegant and concise fashion, this book presents the concepts of functional analysis required by students of mathematics and physics. It begins with the basics of normed linear spaces and quickly proceeds to concentrate on Hilbert spaces, specifically the spectral theorem for bounded as well as unbounded operators in separable Hilbert spaces. While the first two chapters are devoted to basic propositions concerning normed vector spaces and Hilbert spaces, the third chapter treats advanced topics which are perhaps not standard in a first course on functional analysis. It begins with the Gelfand theory of commutative Banach algebras, and proceeds to the Gelfand-Naimark theorem on commutative C^* -algebras. A discussion of representations of C^* -algebras follows, and the final section of this chapter is devoted to the Hahn-Hellinger classification of separable representations of commutative C^* -algebras. After this detour into operator algebras, the fourth chapter reverts to more standard operator theory in Hilbert space, dwelling on topics such as the spectral theorem for normal operators, the polar decomposition theorem, and the Fredholm theory for compact operators. A brief introduction to the theory of unbounded operators on Hilbert space is given in the fifth and final chapter. There is a voluminous appendix whose purpose is to fill in possible gaps in the reader's background in various areas such as linear algebra, topology, set theory and measure theory. The book is interspersed with many exercises, and hints are provided for the solutions to the more challenging of these.

Includes sections on the spectral resolution and spectral representation of self adjoint operators, invariant subspaces, strongly continuous one-parameter semigroups, the index of operators, the trace formula of Lidskii, the Fredholm determinant, and more. * Assumes prior knowledge of Naive set theory, linear algebra, point set topology, basic complex variable, and real variables. * Includes an appendix on the Riesz representation theorem.

Exercises in Analysis will be published in two volumes. This first volume covers problems in five core topics of mathematical analysis: metric spaces; topological spaces; measure, integration and Martingales; measure and topology and functional analysis. Each of five topics correspond to a different chapter with inclusion of the basic theory and accompanying main definitions and results, followed by suitable comments and remarks for better understanding of the material. At least 170 exercises/problems are presented for each topic, with solutions available at the end of each chapter. The entire collection of exercises offers a balanced and useful picture for the application surrounding each topic. This nearly encyclopedic coverage of exercises in mathematical analysis is the first of its kind and is accessible to a wide readership. Graduate students will find the collection of problems valuable in preparation for their preliminary or qualifying exams as well as for testing their deeper understanding of the material. Exercises are denoted by degree of difficulty. Instructors teaching courses that include one or all of the above-mentioned topics will find the exercises of great help in course preparation. Researchers in analysis may find this Work useful as

a summary of analytic theories published in one accessible volume.

A novel, practical introduction to functional analysis In the twenty years since the first edition of Applied Functional Analysis was published, there has been an explosion in the number of books on functional analysis. Yet none of these offers the unique perspective of this new edition. Jean-Pierre Aubin updates his popular reference on functional analysis with new insights and recent discoveries—adding three new chapters on set-valued analysis and convex analysis, viability kernels and capture basins, and first-order partial differential equations. He presents, for the first time at an introductory level, the extension of differential calculus in the framework of both the theory of distributions and set-valued analysis, and discusses their application for studying boundary-value problems for elliptic and parabolic partial differential equations and for systems of first-order partial differential equations. To keep the presentation concise and accessible, Jean-Pierre Aubin introduces functional analysis through the simple Hilbertian structure. He seamlessly blends pure mathematics with applied areas that illustrate the theory, incorporating a broad range of examples from numerical analysis, systems theory, calculus of variations, control and optimization theory, convex and nonsmooth analysis, and more. Finally, a summary of the essential theorems as well as exercises reinforcing key concepts are provided. Applied Functional Analysis, Second Edition is an excellent and timely resource for both pure and applied mathematicians.

Functional analysis owes much of its early impetus to problems that arise in the calculus of variations. In turn, the methods developed there have been applied to optimal control, an area that also requires new tools, such as nonsmooth analysis. This self-contained textbook gives a complete course on all these topics. It is written by a leading specialist who is also a noted expositor. This book provides a thorough introduction to functional analysis and includes many novel elements as well as the standard topics. A short course on nonsmooth analysis and geometry completes the first half of the book whilst the second half concerns the calculus of variations and optimal control. The author provides a comprehensive course on these subjects, from their inception through to the present. A notable feature is the inclusion of recent, unifying developments on regularity, multiplier rules, and the Pontryagin maximum principle, which appear here for the first time in a textbook. Other major themes include existence and Hamilton-Jacobi methods. The many substantial examples, and the more than three hundred exercises, treat such topics as viscosity solutions, nonsmooth Lagrangians, the logarithmic Sobolev inequality, periodic trajectories, and systems theory. They also touch lightly upon several fields of application: mechanics, economics, resources, finance, control engineering. Functional Analysis, Calculus of Variations and Optimal Control is intended to support several different courses at the first-year or second-year graduate level, on functional analysis, on the calculus of variations and optimal control, or on some combination. For this reason, it has been organized with customization in mind.

The text also has considerable value as a reference. Besides its advanced results in the calculus of variations and optimal control, its polished presentation of certain other topics (for example convex analysis, measurable selections, metric regularity, and nonsmooth analysis) will be appreciated by researchers in these and related fields.

The goal of this textbook is to provide an introduction to the methods and language of functional analysis, including Hilbert spaces, Fredholm theory for compact operators, and spectral theory of self-adjoint operators. It also presents the basic theorems and methods of abstract functional analysis and a few applications of these methods to Banach algebras and the theory of unbounded self-adjoint operators. The text corresponds to material for two semester courses (Part I and Part II, respectively), and it is as self-contained as possible. The only prerequisites for the first part are minimal amounts of linear algebra and calculus. However, for the second course (Part II), it is useful to have some knowledge of topology and measure theory. Each chapter is followed by numerous exercises, whose solutions are given at the end of the book.

This book constitutes a concise introductory course on Functional Analysis for students who have studied calculus and linear algebra. The topics covered are Banach spaces, continuous linear transformations, Frechet derivative, geometry of Hilbert spaces, compact operators, and distributions. In addition, the book includes selected applications of functional analysis to differential equations, optimization, physics (classical and quantum mechanics), and numerical analysis. The book contains 197 problems, meant to reinforce the fundamental concepts. The inclusion of detailed solutions to all the exercises makes the book ideal also for self-study. A Friendly Approach to Functional Analysis is written specifically for undergraduate students of pure mathematics and engineering, and those studying joint programmes with mathematics. Request Inspection Copy

This book is based on lectures given at "Mekhmat", the Department of Mechanics and Mathematics at Moscow State University, one of the top mathematical departments worldwide, with a rich tradition of teaching functional analysis. Featuring an advanced course on real and functional analysis, the book presents not only core material traditionally included in university courses of different levels, but also a survey of the most important results of a more subtle nature, which cannot be considered basic but which are useful for applications. Further, it includes several hundred exercises of varying difficulty with tips and references. The book is intended for graduate and PhD students studying real and functional analysis as well as mathematicians and physicists whose research is related to functional analysis.

This book provides an introduction to the ideas and methods of linear functional analysis at a level appropriate to the final year of an undergraduate course at a British university. The prerequisites for reading it are a standard undergraduate knowledge of linear algebra and real analysis (including the theory of metric

spaces). Part of the development of functional analysis can be traced to attempts to find a suitable framework in which to discuss differential and integral equations. Often, the appropriate setting turned out to be a vector space of real or complex-valued functions defined on some set. In general, such a vector space is infinite-dimensional. This leads to difficulties in that, although many of the elementary properties of finite-dimensional vector spaces hold in infinite dimensional vector spaces, many others do not. For example, in general infinite dimensional vector spaces there is no framework in which to make sense of analytic concepts such as convergence and continuity. Nevertheless, on the spaces of most interest to us there is often a norm (which extends the idea of the length of a vector to a somewhat more abstract setting). Since a norm on a vector space gives rise to a metric on the space, it is now possible to do analysis in the space. As real or complex-valued functions are often called functionals, the term functional analysis came to be used for this topic. We now briefly outline the contents of the book.

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