Euclidean And Non Geometries Greenberg Solutions

One of the challenges many mathematics students face occurs after they complete their study of basic calculus and linear algebra, and they start taking courses where they are expected to write proofs. Historically, students have been learning to think mathematically and to write proofs by studying Euclidean geometry. In the author's opinion, geometry is still the best way to make the transition from elementary to advanced mathematics. The book begins with a thorough review of high school geometry, then goes on to discuss special points associated with triangles, circles and certain associated lines, Ceva's theorem, vector techniques of proof, and compass-and-straightedge constructions. There is also some emphasis on proving numerical formulas like the laws of sines, cosines, and tangents, Stewart's theorem, Ptolemy's theorem, and the area formula of Heron. An important difference of this book from the majority of modern college geometry texts is that it avoids axiomatics. The students using this book have had very little experience with formal mathematics. Instead, the focus of the course and the book is on interesting theorems and on the techniques that can be used to prove them. This makes the book suitable to second- or third-year mathematics majors and also to secondary mathematics education majors, allowing the students to learn how to write proofs of mathematical results and, at the end, showing them what mathematics is really all about.
This book is intended as a first rigorous course in geometry. As the title indicates, we have adopted Birkhoff's metric approach (i.e., through use of real numbers) rather than Hilbert's synthetic approach to the subject. Throughout the text we illustrate the various axioms, definitions, and theorems with models ranging from the familiar Cartesian plane to the

Poincare upper half plane, the Taxicab plane, and the Moulton plane. We hope that through an intimate acquaintance with examples (and a model is just an example), the reader will obtain a real feeling and intuition for non Euclidean (and in particular, hyperbolic) geometry. From a pedagogical viewpoint this approach has the advantage of reducing the reader's tendency to reason from a picture. In addition, our students have found the strange new world of the non-Euclidean geometries both interesting and exciting. Our basic approach is to introduce and develop the various axioms slowly, and then, in a departure from other texts, illustrate major definitions and axioms with two or three models. This has the twin advantages of showing the richness of the concept being discussed and of enabling the reader to picture the idea more clearly. Furthermore, encountering models which do not satisfy the axiom being introduced or the hypothesis of the theorem being proved often sheds more light on the relevant concept than a myriad of cases which do.

College-level text for elementary courses covers the fifth postulate, hyperbolic plane geometry and trigonometry, and elliptic plane geometry and trigonometry. Appendixes offer background on Euclidean geometry. Numerous exercises. 1945 edition.

LOBACHEVSKY was the first man ever to publish a non-Euclidean geometry. Of the immortal essay now first appearing in English Gauss said, "The author has treated the matter with a master-hand and in the true geometer's spirit. I think I ought to call your attention to this book, whose perusal cannot fail to give you the most vivid pleasure." Clifford says, "It is quite simple, merely Euclid without the vicious assumption, but the way things come out of one another is quite lovely." * * * "What Vesalius was to Galen, what Copernicus was to Ptolemy, that was Lobachevsky to Euclid." Says Sylvester, "In Quaternions the example has been given of

Algebra released from the yoke of the commutative principle of multiplication - an emancipation somewhat akin to Lobachevsky's of Geometry from Euclid's noted empirical axiom." Cayley says, "It is well known that Euclid's twelfth axiom, even in Playfair's form of it, has been considered as needing demonstration; and that Lobachevsky constructed a perfectly consistent theory, where- in this axiom was assumed not to hold good, or say a system of non-Euclidean plane geometry. There is a like system of non-Euclidean solid geometry." GEORGE BRUCE HALSTED. 2407 San Marcos Street, Austin, Texas. * * * *From the TRANSLATOR'S INTRODUCTION. "Prove all things, hold fast that which is good," does not mean demonstrate everything. From nothing assumed, nothing can be proved. "Geometry without axioms," was a book which went through several editions, and still has historical value. But now a volume with such a title would, without opening it, be set down as simply the work of a paradoxer. The set of axioms far the most influential in the intellectual history of the world was put together in Egypt; but really it owed nothing to the Egyptian race, drew nothing from the boasted lore of Egypt's priests. The Papyrus of the Rhind, belonging to the British Museum, but given to the world by the erudition of a German Egyptologist, Eisenlohr, and a German historian of mathematics, Cantor, gives us more knowledge of the state of mathematics in ancient Egypt than all else previously accessible to the modern world. Its whole testimony con- firms with overwhelming force the position that Geometry as a science, strict and self-conscious deductive reasoning, was created by the subtle intellect of the same race whose bloom in art still overawes us in the Venus of Milo, the Apollo Belvidere, the Laocoon. In a geometry occur the most noted set of axioms, the geometry of Euclid, a pure Greek, professor at the University of Alexandria. Not only at its very birth did this typical product of the Greek genius assume sway as ruler in the pure sciences, not only does its first efflorescence carry us through the splendid days of Theon and Hypatia, but unlike the latter, fanatics cannot murder it; that dismal flood, the dark ages, cannot drown it. Like the phoenix of its native Egypt, it rises with the new birth of culture. An Anglo-Saxon, Adelard of Bath, finds it clothed in Arabic vestments in the land of the Alhambra. Then clothed in Latin, it and the new-born printing press confer honor on each other. Finally back again in its original Greek, it is published first in queenly Basel, then in stately Oxford. The latest edition in Greek is from Leipsic's learned presses.

The story of geometry is the story of mathematics itself: Euclidean geometry was the first branch of mathematics to be systematically studied and placed on a firm logical foundation, and it is the prototype for the axiomatic method that lies at the foundation of modern mathematics. It has been taught to students for more than two millennia as a mode of logical thought. This book tells the story of how the axiomatic method has progressed from Euclid's time to ours, as a way of understanding what mathematics is, how we read and evaluate mathematical arguments, and why mathematics has achieved the level of certainty it has. It is designed primarily for advanced undergraduates who plan to teach secondary school geometry, but it should also provide something of interest to anyone who wishes to understand geometry and the axiomatic method better. It introduces a modern, rigorous, axiomatic treatment of Euclidean and (to a lesser extent) non-Euclidean geometries, offering students ample opportunities to practice reading and writing proofs while at the same time developing most of the concrete geometric relationships that secondary teachers will need to know in the classroom. -- P. [4] of cover.

College Geometry is an approachable text, covering both Euclidean and Non-Euclidean geometry. This text is directed at the one semester course at the college level, for both pure mathematics majors and prospective teachers. A primary focus is on student participation, which is promoted in two ways: (1) Each section of the book contains one or two units, called Moments for Discovery, that use drawing, computational, or reasoning experiments to guide students to an often surprising conclusion related to section concepts; and (2) More than 650 problems were carefully designed to maintain student interest.

Martin Gardner's Mathematical Games columns in Scientific American inspired and entertained several generations of mathematicians and scientists. Gardner in his crystal-clear prose illuminated corners of mathematics, especially recreational mathematics, that most people had no idea existed. His playful spirit and inquisitive nature invite the reader into an exploration of beautiful mathematical ideas along with him. These columns were both a revelation and a gift when he wrote them; no one-before Gardner-had written about mathematics like this. They continue to be a marvel. This is the original 1997 edition and contains columns published from 1980-1986.

Algebraic Topology and basic homotopy theory form a fundamental building block for much of modern mathematics. These lecture notes represent a culmination of many years of leading a two-semester course in this subject at MIT. The style is engaging and student-friendly, but precise. Every lecture is accompanied by exercises. It begins slowly in order to gather up students with a variety of backgrounds, but gains pace as the course progresses, and by the end the student has a command of all the basic techniques of classical homotopy theory.

This book gives a rigorous treatment of the fundamentals of plane geometry: Euclidean, spherical, elliptical and hyperbolic.

Euclidean and Non-Euclidean Geometries presents the discovery of non-Euclidean geometry and the reformulation of the foundations of Euclidean geometry.

This volume presents an accessible, self-contained survey of topics in Euclidean and non-Euclidean geometry. It includes plentiful illustrations and exercises in support of the thoroughly worked-out proofs. The author's emphasis on the connections between Euclidean and non-Euclidean geometry unifies the range of topics covered. The text opens with a brief review of elementary geometry before proceeding to advanced material. Topics covered include advanced Euclidean and non-Euclidean geometry, division ratios and triangles, transformation geometry, projective geometry, conic sections, and hyperbolic and absolute geometry. Topics in Geometry includes over 800 illustrations and extensive exercises of varying difficulty.

What does it mean to know mathematics? How does meaning in mathematics education connect to common sense or to the meaning of mathematics itself? How are meanings constructed and communicated and what are the dilemmas related to these processes? There are many answers to these questions, some of which might appear to be contradictory. Thus understanding the complexity of meaning in mathematics education is a matter of huge importance. There are twin directions in which discussions have developed—theoretical and practical—and this book seeks to move the debate forward along both dimensions while seeking to relate them where appropriate. A discussion of meaning can start from a theoretical examination of mathematics and how mathematicians over time have made sense of their work. However, from a more practical perspective, anybody involved in teaching mathematics is faced with the need to orchestrate the myriad of meanings derived from multiple sources that students develop of mathematical knowledge. This book presents a wide variety of theoretical reflections and research results about meaning in mathematics and mathematics education based on long-term and collective reflection by the group of authors as a whole. It is the outcome of the work of the BACOMET (BAsic COmponents of Mathematics Education for Teachers) group who spent several years deliberating on this topic. The ten chapters in this book, both separately and together, provide a substantial contribution to clarifying the complex issue of meaning in mathematics education. This book is of interest to researchers in mathematics education, graduate students of mathematics education, under graduate students in mathematics, secondary mathematics teachers and primary teachers with an interest in mathematics.

This volume constitutes the refereed proceedings of the 19th International Symposium on Graph Drawing, GD 2010, held in Eindhoven, The Netherlands, during September 2011. The 34 revised full papers presented together with 3 revised short and 6 poster papers were carefully reviewed and selected from 88 submissions. Furthermore, the proceedings contain the abstracts of two invited talks and to commemorate Kozo Sugiyama and his pioneering research in graph drawing, the proceedings include an obituary. A unique and fun part of the symposium is the Graph Drawing Contest, which is part of the Graph Drawing Challenge. This year was the 18th edition. A report on the contest is included at the end of the proceedings.

This remarkable book has endured as a true masterpiece of mathematical exposition. There are few mathematics books that are still so widely read and continue to have so much to offer—even after more than half a century has passed! The book is overflowing with mathematical ideas, which are always explained clearly and elegantly, and above all, with penetrating insight. It is a joy to read, both for beginners and experienced mathematicians.

“Hilbert and Cohn-Vossen” is full of interesting facts, many of which you wish you had known before. It's also likely that you have heard those facts before, but surely wondered where they could be found. The book begins with examples of the simplest curves and surfaces, including thread constructions of certain quadrics and other surfaces. The chapter on regular systems of points leads to the crystallographic groups and the regular polyhedra in R 3 R3. In this chapter, they also discuss plane lattices. By considering unit lattices, and throwing in a small amount of number theory when necessary, they effortlessly derive Leibniz's series: ?/4=1?1/3+1/5?1/7+?… ?/4=1?1/3+1/5?1/7+?…. In the section on lattices in three and more dimensions, the authors consider sphere-packing problems, including the famous Kepler problem. One of the most remarkable chapters is “Projective Configurations”. In a short introductory section, Hilbert and Cohn-Vossen give perhaps the most concise and lucid description of why a general geometer would care about projective geometry and why such an ostensibly plain setup is truly rich in structure and ideas. Here, we see regular polyhedra again, from a different perspective. One of the high points of the chapter is the discussion of Schlafli's Double-Six, which leads to the description of the 27 lines on the general smooth cubic surface. As is true throughout the book, the magnificent drawings in this chapter immeasurably help the reader. A particularly intriguing section in the chapter on differential geometry is Eleven Properties of the Sphere. Which eleven properties of such a ubiquitous mathematical object caught their discerning eye and why? Many mathematicians are familiar with the plaster models of surfaces found in many mathematics departments. The book includes pictures of some of the models that are found in the Göttingen collection. Furthermore, the mysterious lines that mark these surfaces are finally explained! The chapter on kinematics includes a nice discussion of linkages and the geometry of configurations of points and rods that are connected and, perhaps, constrained in some way. This topic in geometry has become increasingly important in recent times, especially in applications to robotics. This is another example of a simple situation that leads to a rich geometry. It would be hard to overestimate the continuing influence Hilbert-Cohn-Vossen's book has had on mathematicians of this century. It surely belongs in the “pantheon” of great mathematics books.

This book is an introduction to hyperbolic and differential geometry that provides material in the early chapters that can serve as a textbook for a standard upper division course on hyperbolic geometry. For that material, the students need to be familiar with calculus and linear algebra and willing to accept one advanced theorem from analysis without proof. The book goes well beyond the standard course in later chapters, and there is enough material for an honors course, or for supplementary reading. Indeed, parts of the book have been used for both kinds of courses. Even some of what is in the early chapters would surely not be nec essary for a standard course. For example, detailed proofs are given of the Jordan Curve Theorem for Polygons and of the decomposability of poly gons into triangles, These proofs are included for the sake of completeness, but the results themselves are so believable that most students should skip the proofs on a first reading. The axioms used are modern in character and more "user friendly" than the traditional ones. The familiar real number system is used as an in gredient rather than appearing as a result of the axioms. However, it should not be thought that the geometric treatment is in terms of models: this is an axiomatic approach that is just more convenient than the traditional ones.

This book presents the reader with a comprehensive overview of the major findings of the recent research on the illusion of linearity. It discusses: how the illusion of linearity appears in diverse domains of mathematics and science; what are the crucial psychological, mathematical, and educational factors being responsible for the occurrence and persistence of the phenomenon; and how the illusion of linearity can be remedied.

Euclidean and Non-Euclidean GeometriesDevelopment and HistoryMacmillan

This first complete English language edition of Euclides vindicatus presents a corrected and revised edition of the classical English translation of Saccheri's text by G.B. Halsted. It is complemented with a historical introduction on the geometrical environment of the time and a detailed commentary that helps to understand the aims and subtleties of the work. Euclides vindicatus, written by the Jesuit mathematician Gerolamo Saccheri, was published in Milan in 1733. In it, Saccheri attempted to reform elementary geometry in two important directions: a demonstration of the famous Parallel Postulate and the theory of proportions. Both topics were of pivotal importance in the mathematics of the time. In particular, the Parallel Postulate had escaped demonstration since the first attempts at it in the Classical Age, and several books on the topic were published in the Early Modern Age. At the same time, the theory of proportion was the most important mathematical tool of the Galilean School in its pursuit of the mathematization of nature. Saccheri's attempt to prove the Parallel Postulate is today considered the most important breakthrough in geometry in the 18th century, as he was able to develop for hundreds of pages and dozens of theorems a system in geometry that denied the truth of the postulate (in the attempt to find a contradiction). This can be regarded as the first system of non-Euclidean geometry. Its later developments by Lambert, Bolyai, Lobachevsky and Gauss eventually opened the way to contemporary geometry. Occupying a unique position in the literature of mathematical history, Euclid Vindicated from Every Blemish will be of high interest to historians of mathematics as well as historians of philosophy interested in the development of non-Euclidean geometries.

Brings to life many of the characters who played a role in the development of the Pythagorean theorem--from the ancient Babylonians and Pythagoras to Albert Einstein and modern-day mathematicians--in a history that provides a fascinating backdrop to an enduring mathematical legacy.

Mathematics is a poem. It is a lucid, sensual, precise exposition of beautiful ideas directed to specific goals. It is worthwhile to have as broad a cross-section of mankind as possible be conversant with what goes on in mathematics. Just as everyone knows that the Internet is a powerful and important tool for communication, so everyone should know that the Poincaré conjecture gives us important information about the shape of our universe. Just as every responsible citizen realizes that the massproduction automobile was pioneered by Henry Ford, so everyone should know that the P/NP problem has implications for security and data manipulation that will affect everyone. This book endeavors to tell the story of the modern impact of mathematics, of its trials and triumphs and insights, in language that can be appreciated by a broad audience. It endeavors to show what mathematics means for our lives, how it impacts all of us, and what new thoughts it should cause us to entertain. It introduces new vistas of mathematical ideas and shares the excitement of new ideas freshly minted. It discusses the significance and impact of these ideas, and gives them meaning that will travel well and cause people to reconsider their place in the universe. Mathematics is one of mankind's oldest disciplines. Along with philosophy, it has shaped the very modus of human thought. And it continues to do so. To be unaware of modern mathematics is to miss out on a large slice of life. It is to be left out of essential modern developments. We want to address this point, and do something about it. This is a book to make mathematics exciting for people of all interests and all walks of life. Mathematics is exhilarating, it is ennobling, it is uplifting, and it is fascinating. We want to show people this part of our world, and to get them to travel new paths.

Great first book on algebraic topology. Introduces (co)homology through singular theory.

Never HIGHLIGHT a Book Again! Virtually all of the testable terms, concepts, persons, places, and events from the textbook are included. Cram101 Just the FACTS101 studyguides give all of the outlines, highlights, notes, and quizzes for your textbook with optional online comprehensive practice tests. Only Cram101 is Textbook Specific. Accompanys: 9780716799481 .

This text promotes student engagement with the beautiful ideas of geometry. Every major concept is introduced in its historical context and connects the idea with real-life. A system of experimentation followed by rigorous explanation and proof is central. Exploratory projects play an integral role in this text. Students develop a better sense of how to prove a result and visualize connections between statements, making these connections real. They develop the intuition needed to conjecture a theorem and devise a proof of what they have observed.

Introduction to vector algebra in the plane; circles and coaxial systems; mappings of the Euclidean plane; similitudes, isometries, Moebius transformations, much more. Includes over 500 exercises.

The name non-Euclidean was used by Gauss to describe a system of geometry which differs from Euclid's in its properties of parallelism. Such a system was developed independently by Bolyai in Hungary and Lobatschewsky in Russia, about 120 years ago. Another system, differing more radically from Euclid's, was suggested later by Riemann in Germany and Cayley in England. The subject was unified in 1871 by Klein, who gave the names of parabolic, hyperbolic, and elliptic to the respective systems of Euclid-Bolyai-Lobatschewsky, and Riemann-Cayley. Since then, a vast literature has accumulated. The Fifth edition adds a new chapter, which includes a description of the two families of 'mid-lines' between two given lines, an elementary derivation of the basic formulae of spherical trigonometry and hyperbolic trigonometry, a computation of the Gaussian curvature of the elliptic and hyperbolic planes, and a proof of Schlafli's remarkable formula for the differential of the volume of a tetrahedron.

Lively guide by a prominent historian focuses on the role of Euclid's Elements in subsequent mathematical developments. Elementary algebra and plane geometry are sole prerequisites. 80 drawings. 1963 edition.

"There is perhaps no better way to prepare for the scientific breakthroughs of tomorrow than to learn the language of geometry." -Brian Greene, author of The Elegant Universe The word "geometry" brings to mind an array of mathematical images: circles, triangles, the Pythagorean Theorem. Yet geometry is so much more than shapes and numbers; indeed, it governs much of our lives-from architecture and microchips to car design, animated movies, the molecules of food, even our own body chemistry. And as Siobhan Roberts elegantly conveys in The King of Infinite Space, there can be no better guide to the majesty of geometry than Donald Coxeter, perhaps the greatest geometer of the twentieth century. Many of the greatest names in intellectual history-Pythagoras, Plato, Archimedes, Euclid- were geometers, and their creativity and achievements illuminate those of Coxeter, revealing geometry to be a living, ever-evolving endeavor, an intellectual adventure that has always been a building block of civilization. Coxeter's special contributions-his famed Coxeter groups and Coxeter diagrams-have been called by other mathematicians "tools as essential as numbers themselves," but his greatest achievement was to almost single-handedly preserve the tradition of classical geometry when it was under attack in a mathematical era that valued all things austere and rational. Coxeter also inspired many outside the field of mathematics. Artist M. C. Escher credited Coxeter with triggering his legendary Circle Limit patterns, while futurist/inventor Buckminster Fuller acknowledged that his famed geodesic dome owed much to Coxeter's vision. The King of Infinite Space is an elegant portal into the fascinating, arcane world of geometry.

Richly detailed survey of the evolution of geometrical ideas and development of concepts of modern geometry: projective, Euclidean, and non-Euclidean geometry; role of geometry in Newtonian physics, calculus, relativity. Over 100 exercises with answers. 1966 edition.

Geometry was considered until modern times to be a model science. To be developed more geometrico was a seal of quality for any endeavor, whether mathematical or not. In the 17th century, for example, Spinoza set up his Ethics in a more geometrico manner, to emphasize the perfection, certainty, and clarity of his pronouncements. Geometry achieved this status on the heels of Euclid's Elements, in which, for the first time, a theory was built up in an axiomatic-deductive manner. Euclid started with obvious axioms - he called them "common notions" and "postulates" -, statements whose validity raised no doubts in the reader's mind. His propositions followed deductively from those axioms, so that the truth of the axioms was passed on to the propositions by means of purely logical proofs. In this sense, Euclid's geometry consisted of "eternal truths." Given its prominence, Euclid's Elements was also used as a textbook until the 20th Century. Today geometry has lost the central importance it had during earlier centuries, but it still is an important area of mathematics, and is truly fundamental for mathematics from a variety of points of view. The "Introduction to Geometry" by Ewald tries to address some of these points of view, whose significance will be examined in what follows from a historical perspective.

Geometry has been an essential element in the study of mathematics since antiquity. Traditionally, we have also learned formal reasoning by studying Euclidean geometry. In this book, David Clark develops a modern axiomatic approach to this ancient subject, both in content and presentation. Mathematically, Clark has chosen a new set of axioms that draw on a modern understanding of set theory and logic, the real number continuum and measure theory, none of which were available in Euclid's time. The result is a development of the standard content of Euclidean geometry with the mathematical precision of Hilbert's foundations of geometry. In particular, the book covers all the topics listed in the Common Core State Standards for high school synthetic geometry. The presentation uses a guided inquiry, active learning pedagogy. Students benefit from the axiomatic development because they themselves solve the problems and prove the theorems with the instructor serving as a guide and mentor. Students are thereby empowered with the knowledge that they can solve problems on their own without reference to authority. This book, written for an undergraduate axiomatic geometry course, is particularly well suited for future secondary school teachers. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.

This classic text provides overview of both classic and hyperbolic geometries, placing the work of key mathematicians/ philosophers in historical context. Coverage includes geometric transformations, models of the hyperbolic planes, and pseudospheres.

Thinking Geometrically: A Survey of Geometries is a well written and comprehensive survey of college geometry that would serve a wide variety of courses for both mathematics majors and mathematics education majors. Great care and attention is spent on developing visual insights and geometric intuition while stressing the logical structure, historical development, and deep interconnectedness of the ideas. Students with less mathematical preparation than upper-division mathematics majors can successfully study the topics needed for the preparation of high school teachers. There is a multitude of exercises and projects in those chapters developing all aspects of geometric thinking for these students as well as for more advanced students. These chapters include Euclidean Geometry, Axiomatic Systems and Models, Analytic Geometry, Transformational Geometry, and Symmetry. Topics in the other chapters, including Non-Euclidean Geometry, Projective Geometry, Finite Geometry, Differential Geometry, and Discrete Geometry, provide a broader view of geometry. The different chapters are as independent as possible, while the text still manages to highlight the many connections between topics. The text is self-contained, including appendices with the material in Euclid’s first book and a high school axiomatic system as well as Hilbert’s axioms. Appendices give brief summaries of the parts of linear algebra and multivariable calculus needed for certain chapters. While some chapters use the language of groups, no prior experience with abstract algebra is presumed. The text will support an approach emphasizing dynamical geometry software without being tied to any particular software.

Presented as an engaging discourse, this textbook invites readers to delve into the historical origins and uses of geometry. The narrative traces the influence of Euclid's system of geometry, as developed in his classic text The Elements, through the Arabic period, the modern era in the West, and up to twentieth century mathematics. Axioms and proof methods used by mathematicians from those periods are explored alongside the problems in Euclidean geometry that lead to their work. Students cultivate skills applicable to much of modern mathematics through sections that integrate concepts like projective and hyperbolic geometry with representative proof-based exercises. For its sophisticated account of ancient to modern geometries, this text assumes only a year of college mathematics as it builds towards its conclusion with algebraic curves and quaternions. Euclid's work has affected geometry for thousands of years, so this text has something to offer to anyone who wants to broaden their appreciation for the field.

This book offers a unique opportunity to understand the essence of one of the great thinkers of western civilization. A guided reading of Euclid's Elements leads to a critical discussion and rigorous modern treatment of Euclid's geometry and its more recent descendants, with complete proofs. Topics include the introduction of coordinates, the theory of area, history of the parallel postulate, the various non-Euclidean geometries, and the regular and semi-regular polyhedra.