Decomposition Methods For Differential Equations Theory And Applications Chapman Hallcrc Numerical Analysis And Scientific Computing Series

The purpose of this book is to offer an overview of the most popular domain decomposition methods for partial differential equations (PDEs). These methods are widely used for numerical simulations in solid mechanics, electromagnetism, flow in porous media, etc., on parallel machines from tens to hundreds of thousands of cores. The appealing feature of domain decomposition methods is that, contrary to direct methods, they are naturally parallel. The authors focus on parallel linear solvers. The authors present all popular algorithms, both at the PDE level and at the discrete level in terms of matrices, along with systematic scripts for sequential implementation in a free open-source finite element package as well as some parallel scripts. Also included is a new coarse space construction (two-level method) that adapts to highly heterogeneous problems.÷

This refereed volume arose from the editors' recognition that physical scientists, engineers, and applied mathematicians are developing, in parallel, solutions to problems of parallelization. The cross-disciplinary field of scientific computation is bringing about better communication between heterogeneous computational groups, as they face this common challenge. This volume is one attempt to provide cross-disciplinary communication. Problem decomposition and the use of domain-based parallelism in computational science and engineering was the subject addressed at a workshop held at the University of Minnesota Supercomputer Institute in April 1994. The authors were subsequently able to address the relationships between their individual applications and independently developed approaches. This book is written for an interdisciplinary audience and concentrates on transferable algorithmic techniques, rather than the scientific results themselves. Cross-disciplinary editing was employed to identify jargon that needed further explanation and to ensure provision of a brief scientific background for each chapter at a tutorial level so that the physical significance of the variables is clear and correspondences between fields are visible.

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Targeted at students and researchers in computational sciences who need to develop computer codes for solving PDEs, the exposition here is focused on numerics and software related to mathematical models in solid and fluid mechanics. The book teaches finite element methods, and basic finite difference methods from a computational point of view, with the main emphasis on developing flexible computer programs, using the numerical library Diffpack. Diffpack is explained in detail for problems including model equations in applied mathematics, heat transfer, elasticity, and viscous fluid flow. All the program examples, as well as Diffpack for use with this book, are available on the Internet. XXXXXXX NEUER TEXT This book is for researchers who need to develop computer code for solving PDEs. Numerical methods and the application of Diffpack are explained in detail. Diffpack is a modern C++ development environment that is widely used by industrial scientists and engineers working in areas such as oil exploration, groundwater modeling, and materials testing. All the program examples, as well as a test version of Diffpack, are available for free over the Internet.

Harmonic Wave Systems is the first textbook about the computational method of Decomposition in Invariant Structures (DIS) that generalizes the analytical methods of separation of variables, undetermined coefficients, asymptotic expansions, and series expansions. In recent years, there has been a boom in publications on propagation of nonlinear waves described by a fascinating list of partial differential equations (PDEs). The vast majority of wave problems are reducible to one-dimensional ones in propagation variables. However, a list of publications with two- and three-dimensional applications of the DIS method is brief. The book offers a comprehensive and rigorous treatment of the DIS method in two and three dimensions using the PDE approach to the Helmholtz decomposition that provides the most general background for mathematical modelling of harmonic waves in fluid dynamics, electrodynamics, heat transfer, and other numerous areas of science and engineering, which are dealing with propagation and interaction of N internal waves.

Nonlinear differential equations are ubiquitous in computational science and engineering modeling, fluid dynamics, finance, and quantum mechanics, among other areas. Nowadays, solving challenging problems in an industrial setting requires a continuous interplay between the theory of such systems and the development and use of sophisticated computational methods that can guide and support the theoretical findings via practical computer simulations. Owing to the impressive development in computer technology and the introduction of fast numerical methods with reduced algorithmic and memory complexity, rigorous solutions in many applications have become possible. This book collects research papers from leading world experts in the field, highlighting ongoing trends, progress, and open problems in this critically important area of mathematics.

A Powerful Methodology for Solving All Types of Differential Equations Decomposition Analysis Method in Linear and Non-Linear Differential Equations explains how the Adomian decomposition method can solve differential equations for the series solutions of fundamental problems in physics, astrophysics, chemistry, biology, medicine, and other scientific areas. This method is advantageous as it simplifies a real problem to reduce it to a mathematically tractable form. The book covers the four classes of the decomposition method: regular/ordinary decomposition, double decomposition, modified decomposition, and asymptotic decomposition. It applies these classes to Laplace and Navier–Stokes equations in Cartesian and polar coordinates for obtaining partial solutions of the equations. Examples of physical and physiological problems, such as tidal waves in a channel, fluids between plates and through tubes, the flow of blood through arteries, and the flow past a wave-shaped wall, demonstrate the applications. Drawing on the author’s extensive research in fluid and gas dynamics, this book shows how the powerful decomposition methodology of Adomian can solve differential equations in a way comparable to any contemporary superfast computer.

Decomposition Methods for Differential Equations: Theory and Applications describes the analysis of numerical methods for evolution equations based on temporal and spatial decomposition methods. It covers real-life problems, the underlying decomposition and discretization, the stability and consistency analysis of the decomposition methods, and numerical results. The book focuses on the modeling of selected multi-physics problems, before introducing decomposition analysis. It presents time and space discretization, temporal decomposition, and the combination of time and spatial decomposition methods for parabolic and hyperbolic equations. The author then applies these methods to numerical problems, including test examples and real-world problems in physical and engineering applications. For the computational results, he uses various software tools, such as MATLAB®, R3T, WIAS-HiTNIHS, and OPERA-SPLITT. Exploring iterative operator-splitting methods, this book shows how to use higher-order discretization methods to solve differential equations. It discusses decomposition methods and their effectiveness, combination possibility with discretization methods, multi-scaling possibilities, and stability to initial and boundary values problems.

Domain decomposition methods are a well established tool for an efficient numerical solution of partial differential equations, in particular for the coupling of different model equations and of different discretization methods. Based on the approximate solution of local boundary value problems either by finite or boundary element methods, the global problem is reduced to an operator equation on the skeleton of the domain decomposition. Different variational formulations then lead to hybrid domain decomposition methods.

Focuses on the notion that by breaking the domain of the original problem into subdomains, such an approach can, if properly implemented, lead to a considerable speedup. The methods are particularly well suited for parallel computers.

Papers presented at the May 1991 symposium reflect continuing interest in the role of domain decomposition in the effective utilization of parallel systems; applications in fluid mechanics, structures, biology, and design optimization; and maturation of analysis of elliptic equations, with theoretic

In this text, we introduce the basic concepts for the numerical modelling of partial differential equations. We consider the classical elliptic, parabolic and hyperbolic linear equations, but also the diffusion, transport, and Navier-Stokes equations, as well as equations representing conservation laws, saddle-point problems and optimal control problems. Furthermore, we provide numerous physical examples which underline such equations. We then analyze numerical solution methods based on finite elements, finite differences, finite volumes, spectral methods and domain decomposition methods, and reduced basis methods. In particular, we discuss the algorithmic and computer implementation aspects and provide a number of easy-to-use programs. The text does not require any previous advanced mathematical knowledge of partial differential equations: the absolutely essential concepts are reported in a preliminary chapter. It is therefore suitable for students of bachelor and master courses in scientific disciplines, and recommendable to those researchers in the academic and extra-academic domain who want to approach this interesting branch of applied mathematics.

This book discusses various novel analytical and numerical methods for solving partial and fractional differential equations. Moreover, it presents selected numerical methods for solving stochastic point kinetic equations in nuclear reactor dynamics by using Euler–Maruyama and strong-order Taylor numerical methods. The book also shows how to arrive at new, exact solutions to various fractional differential equations, such as the time-fractional Burgers–Hopf equation, the (3+1)-dimensional time-fractional Khokhlov–Zabolotskaya–Kuznetsov equation, (3+1)-dimensional time-fractional KdV–Khokhlov–Zabolotskaya–Kuznetsov equation, fractional (2+1)-dimensional Davey–Stewartson equation, and integrable Davey–Stewartson-type equation. Many of the methods discussed are analytical–numerical, namely the modified decomposition method, a new two-step Adomian decomposition method, new approach to the Adomian decomposition method, modified homotopy analysis method with Fourier transform, modified fractional reduced differential transform method (MFRDTM), coupled fractional reduced differential transform method (CFRDTM), optimal homotopy asymptotic method, first integral method, and a solution procedure based on Haar wavelets and the operational matrices with function approximation. The book proposes for the first time a generalized order operational matrix of Haar wavelets, as well as new techniques (MFRDTM and CFRDTM) for solving fractional differential equations. Numerical methods used to solve stochastic point kinetic equations, like the Wiener process, Euler–Maruyama, and order 1.5 strong Taylor methods, are also discussed.

Domain decomposition methods provide powerful and flexible tools for the numerical approximation of partial differential equations arising in the modeling of many interesting applications in science and engineering. This book deals with discretization techniques on non-matching triangulations and iterative solvers with particular emphasis on mortar finite elements, Schwarz methods and multigrid techniques. New results on non-standard situations as mortar methods based on dual basis functions and vector field discretizations are analyzed and illustrated by numerical results. The role of trace theorems, harmonic extensions, dual norms and weak interface conditions is emphasized. Although the original idea was used successfully more than a hundred years ago, these methods are relatively new for the numerical approximation. The possibilites of high performance computations and the interest in large- scale problems have led to an increased research activity.

This book introduces finite difference methods for both ordinary differential equations (ODEs) and partial differential equations (PDEs) and discusses the similarities and differences between algorithm design and stability analysis for different types of equations. A unified view of stability theory for ODEs and PDEs is presented, and the interplay between ODE and PDE analysis is stressed. The text emphasizes standard classical methods, but several newer approaches also are introduced and are described in the context of simple motivating examples.

A domain decomposition method for solving partial differential equations is described. The conditions on interfaces will all be of Dirichlet type and obtained by the boundary element method using very few (less than 10) unknowns. (KR).

Proper Orthogonal Decomposition Methods for Partial Differential Equations evaluates the potential applications of POD reduced-order numerical methods in increasing computational efficiency, decreasing calculating load and alleviating the accumulation of truncation error in the computational process. Introduces the foundations of finite-differences, finite-elements and finite-volume-elements. Models of time-dependent PDEs are presented, with detailed numerical procedures, implementation and error analysis. Output numerical data are plotted in graphics and compared using standard traditional methods. These models contain parabolic, hyperbolic and nonlinear systems of PDEs, suitable for the user to learn and adapt methods to their own R&D problems. Explains ways to reduce order for PDEs by means of the POD method so that reduced-order models have few unknowns Helps readers speed up computation and reduce computation load and memory requirements while numerically capturing system characteristics Enables readers to apply and adapt the methods to solve similar problems for PDEs of hyperbolic, parabolic and nonlinear types

Engineering applications offer benefits and opportunities across a range of different industries and fields. By developing effective methods of analysis, results and solutions are produced with higher accuracy. Numerical and Analytical Solutions for Solving Nonlinear Equations in Heat Transfer is an innovative source of academic research on the optimized techniques for analyzing heat transfer equations and the application of these methods across various fields. Highlighting pertinent topics such as the differential transformation method, industrial applications, and the homotopy perturbation method, this book is ideally designed for engineers, researchers, graduate students, professionals, and academics interested in applying new mathematical techniques in engineering sciences.

"Partial Differential Equations and Solitary Waves Theory" is a self-contained book divided into two parts: Part I is a coherent survey bringing together newly developed methods for solving PDEs. While some traditional techniques are presented, this part does not require thorough understanding of abstract theories or compact concepts. Well-selected worked examples and exercises shall guide the reader through the text. Part II provides an extensive exposition of the solitary waves theory. This part handles nonlinear evolution equations by methods such as Hirota’s bilinear method or the tanh-coth method. A self-contained treatment is presented to discuss complete integrability of a wide class of nonlinear equations. This part presents in an accessible manner a systematic presentation of solitons, multi-soliton solutions, kinks, peakons, cuspons, and compactons. While the whole book can be used as a text for advanced undergraduate and graduate students in applied mathematics, physics and engineering, Part II will be most useful for graduate students and researchers in mathematics, engineering, and other related fields. Dr. Abdul-Majid Wazwaz is a Professor of Mathematics at Saint Xavier University, Chicago, Illinois, USA.

Examines numerical and semi-analytical methods for differential equations that can be used for solving practical ODEs and PDEs This student-friendly book deals with various approaches for solving differential equations numerically or semi-analytically depending on the type of equations and offers simple example problems to help readers along. Featuring both traditional and recent methods, Advanced Numerical and Semi Analytical Methods for Differential Equations begins with a review of basic numerical methods. It then looks at Laplace, Fourier, and weighted residual methods for solving differential equations. A new challenging method of Boundary Characteristics Orthogonal Polynomials (BCOPs) is introduced next. The book then discusses Finite Difference Method (FDM), Finite Element Method (FEM), Finite Volume Method (FVM), and Boundary Element Method (BEM). Following that, analytical/semi analytic methods like Akbari Ganji's Method (AGM) and Exp-function are used to solve nonlinear differential equations. Nonlinear differential equations using semi-analytical methods are also addressed, namely Adomian Decomposition Method (ADM), Homotopy Perturbation Method (HPM), Variational Iteration Method (VIM), and Homotopy Analysis Method (HAM). Other topics covered include: emerging areas of research related to the solution of differential equations based on differential quadrature and wavelet approach; combined and hybrid methods for solving differential equations; as well as an overview of fractal differential equations. Further, uncertainty in term of intervals and fuzzy numbers have also been included, along with the interval finite element method. This book: Discusses various methods for solving linear and nonlinear ODEs and PDEs Covers basic numerical techniques for solving differential equations along with various discretization methods Investigates nonlinear differential equations using semi-analytical methods Examines differential equations in an uncertain environment Includes a new scenario in which uncertainty (in term of intervals and fuzzy numbers) has been included in differential equations Contains solved example problems, as well as some unsolved problems for self-validation of the topics covered Advanced Numerical and Semi Analytical Methods for Differential Equations is an excellent text for graduate as well as post graduate students and researchers studying various methods for solving differential equations, numerically and semi-analytically.

Domain decomposition is an active, interdisciplinary research area that is devoted to the development, analysis and implementation of coupling and decoupling strategies in mathematics, computational science, engineering and industry. A series of international conferences starting in 1987 set the stage for the presentation of many meanwhile classical results on substructuring, block iterative methods, parallel and distributed high performance computing etc. This volume contains a selection from the papers presented at the 15th International Domain Decomposition Conference held in Berlin, Germany, July 17-25, 2003 by the world's leading experts in the field. Its special focus has been on numerical analysis, computational issues,complex heterogeneous problems, industrial problems, and software development.

Fractional calculus provides the possibility of introducing integrals and derivatives of an arbitrary order in the mathematical modelling of physical processes, and it has become a relevant subject with applications to various fields, such as anomalous diffusion, propagation in different media, and propogation in relation to materials with different properties. However, many aspects from theoretical and practical points of view have still to be developed in relation to models based on fractional operators. This Special Issue is related to new developments on different aspects of fractional differential equations, both from a theoretical point of view and in terms of applications in different fields such as physics, chemistry, or control theory, for instance. The topics of the Issue include fractional calculus, the mathematical analysis of the properties of the solutions to fractional equations, the extension of classical approaches, or applications of fractional equations to several fields.

While domain decomposition methods have a long history dating back well over one hundred years, it is only during the last decade that they have become a major tool in numerical analysis of partial differential equations. This monograph emphasizes domain decomposition methods in the context of so-called virtual optimal control problems and treats optimal control problems for partial differential equations and their decompositions using an all-at-once approach.

Going beyond standard introductory texts, Mathematical Optics: Classical, Quantum, and Computational Methods brings together many new mathematical techniques from optical science and engineering research. Profusely illustrated, the book makes the material accessible to students and newcomers to the field. Divided into six parts, the text presents state-of-the-art mathematical methods and applications in classical optics, quantum optics, and image processing. Part I describes the use of phase space concepts to characterize optical beams and the application of dynamic programming in optical waveguides. Part II explores solutions to paraxial, linear, and nonlinear wave equations. Part III discusses cutting-edge areas in transformation optics (such as invisibility cloaks) and computational plasmonics. Part IV uses Lorentz groups, dihedral group symmetry, Lie algebras, and Liouville space to analyze problems in polarization, ray optics, visual optics, and quantum optics. Part V examines the role of coherence functions in modern laser physics and explains how to apply quantum memory channel models in quantum computers. Part VI introduces super-resolution imaging and differential geometric methods in image processing. As numerical/symbolic computation is an important tool for solving numerous real-life problems in optical science, many chapters include Mathematica® code in their appendices. The software codes and notebooks as well as color versions of the book's figures are available at www.crcpress.com.

The book is devoted to recent developments in the theory of fractional calculus and its applications. Particular attention is paid to the applicability of this currently popular research field in various branches of pure and applied mathematics. In particular, the book focuses on the more recent results in mathematical physics, engineering applications, theoretical and applied physics as quantum mechanics, signal analysis, and in those relevant research fields where nonlinear dynamics occurs and several tools of nonlinear analysis are required. Dynamical processes and dynamical systems of fractional order attract researchers from many areas of sciences and technologies, ranging from mathematics and physics to computer science.

This book offers a comprehensive presentation of some of the most successful and popular domain decomposition preconditioners for finite and spectral element approximations of partial differential equations. It places strong emphasis on both algorithmic and mathematical aspects. It covers in detail important methods such as FETI and balancing Neumann-Neumann methods and algorithms for spectral element methods.

Stochastic Systems

Presents an easy-to-read discussion of domain decomposition algorithms, their implementation and analysis. Ideal for graduate students about to embark on a career in computational science. It will also be a valuable resource for all those interested in parallel computing and numerical computational methods.

Domain decomposition methods are divide and conquer computational methods for the parallel solution of partial differential equations of elliptic or parabolic type. The methodology includes iterative algorithms, and techniques for non-matching grid discretizations and heterogeneous approximations. This book serves as a matrix oriented introduction to domain decomposition methodology. A wide range of topics are discussed include hybrid formulations, Schwarz, and many more.

This book presents the first survey of the Localized Orthogonal Decomposition (LOD) method, a pioneering approach for the numerical homogenization of partial differential equations with multiscale data beyond periodicity and scale separation. The authors provide a careful error analysis, including previously unpublished results, and a complete implementation of the method in MATLAB. They also reveal how the LOD method relates to classical homogenization and domain decomposition. Illustrated with numerical experiments that demonstrate the significance of the method, the book is enhanced by a survey of applications including eigenvalue problems and evolution problems. Numerical Homogenization by Localized Orthogonal Decomposition is appropriate for graduate students in applied mathematics, numerical analysis, and scientific computing. Researchers in the field of computational partial differential equations will find this self-contained book of interest, as will applied scientists and engineers interested in multiscale simulation.

Nonlinear Stochastic Operator Equations deals with realistic solutions of the nonlinear stochastic equations arising from the modeling of frontier problems in many fields of science. This book also discusses a wide class of equations to provide modeling of problems concerning physics, engineering, operations research, systems analysis, biology, medicine. This text discusses operator equations and the decomposition method. This book also explains the limitations, restrictions and assumptions made in differential equations involving stochastic process coefficients (the stochastic operator case), which yield results very different from the needs of the actual physical problem. Real-world application of mathematics to actual physical problems, requires making a reasonable model that is both realistic and solvable. The decomposition approach or model is an approximation method to solve a wide range of problems. This book explains an inherent feature of real systems—known as nonlinear behavior—that occurs frequently in nuclear reactors, in physiological systems, or in cellular growth. This text also discusses stochastic operator equations with linear boundary conditions. This book is intended for students with a mathematics background, particularly senior undergraduate and graduate students of advanced mathematics, of the physical or engineering sciences.

The Adomian decomposition method enables the accurate and efficient analytic solution of nonlinear ordinary or partial differential equations without the need to resort to linearization or perturbation approaches. It unifies the treatment of linear and nonlinear, ordinary or partial differential equations, or systems of such equations, into a single basic method, which is applicable to both initial and boundary-value problems. This volume deals with the application of this method to many problems of physics, including some frontier problems which have previously required much more computationally-intensive approaches. The opening chapters deal with various fundamental aspects of the decomposition method. Subsequent chapters deal with the application of the method to nonlinear oscillatory systems in physics, the Duffing equation, boundary-value problems with closed irregular contours or surfaces, and other frontier areas. The potential application of this method to a wide range of problems in diverse disciplines such as biology, hydrology, semiconductor physics, wave propagation, etc., is highlighted. For researchers and graduate students of physics, applied mathematics and engineering, whose work involves mathematical modelling and the quantitative solution of systems of equations.