Convex Sets And Their Applications Dover Books On Mathematics

A gentle introduction to the geometry of convex sets in n-dimensional space Geometry of Convex Sets begins with basic definitions of the concepts of vector addition and scalar multiplication and then defines the notion of convexity for subsets of n-dimensional space. Many properties of convex sets can be discovered using just the linear structure. However, for more interesting results, it is necessary to introduce the notion of distance in order to discuss open sets, closed sets, bounded sets, and compact sets. The book illustrates the interplay between these linear and topological concepts, which makes the notion of convexity so interesting. Thoroughly class-tested, the book discusses topology and convexity in the context of normed linear spaces, specifically with a norm topology on an n-dimensional space. Geometry of Convex Sets also features: An introduction to n-dimensional geometry including points; lines; vectors; distance; norms; inner products; orthogonality; convexity; hyperplanes; and linear functionals Coverage of n-dimensional norm topology including interior points and open sets; accumulation points and closed sets; boundary points and closed sets; compact subsets of n-dimensional space; completeness of n-dimensional space; sequences; equivalent norms; distance between sets; and support hyperplanes · Basic properties of convex sets; convex hulls; interior and closure of convex sets; closed convex hulls; accessibility lemma; regularity of convex sets; affine hulls; flats or affine subspaces; affine basis theorem; separation theorems; extreme points of convex sets; supporting hyperplanes and extreme points; existence of extreme points; Krein–Milman theorem; polyhedral sets and polytopes; and Birkhoff’s theorem on doubly stochastic matrices Discussions of Helly’s theorem; the Art Gallery theorem; Vincensini’s problem; Hadwiger’s theorems; theorems of Radon and Caratheodory; Kirchberger’s theorem; Helly-type theorems for circles; covering problems; piercing problems; sets of constant width; Reuleaux triangles; Barbier’s theorem; and Borsuk’s problem Geometry of Convex Sets is a useful textbook for upper-undergraduate level courses in geometry of convex sets and is essential for graduate-level courses in convex analysis. An excellent reference for academics and readers interested in learning the various applications of convex geometry, the book is also appropriate for teachers who would like to convey a better understanding and appreciation of the field to students. I. E. Leonard, PhD, was a contract lecturer in the Department of Mathematical and Statistical Sciences at the University of Alberta. The author of over 15 peer-reviewed journal articles, he is a technical editor for the Canadian Applied Mathematical Quarterly journal. J. E. Lewis, PhD, is Professor Emeritus in the Department of Mathematical Sciences at the University of Alberta. He was the recipient of the Faculty of Science Award for Excellence in Teaching in 2004 as well as the PIMS Education Prize in 2002.

Convex optimization has an increasing impact on many areas of mathematics, applied sciences, and practical applications. It is now being taught at many universities and being used by researchers of different fields. As convex analysis is the mathematical f

This comprehensive monograph details polynomially convex sets. It presents the general properties of polynomially convex sets with particular attention to the theory of the hulls of one-dimensional sets. Coverage examines in considerable detail questions of uniform approximation for the most part on compact sets but with some attention to questions of global approximation on noncompact sets. The book also discusses important applications and motivates the reader with numerous examples and counterexamples, which serve to illustrate the general theory and to delineate its boundaries.

The study of Euclidean distance matrices (EDMs) fundamentally asks what can be known geometrically given onlydistance information between points in Euclidean space. Each point may represent simply locationor, abstractly, any entity expressible as a vector in finite-dimensional Euclidean space.The answer to the question posed is that very much can be known about the points;the mathematics of this combined study of geometry and optimization is rich and deep.Throughout we cite beacons of historical accomplishment.The application of EDMs has already proven invaluable in discerning biological molecular conformation.The emerging practice of localization in wireless sensor networks, the global positioning system (GPS), and distance-based pattern recognitionwill certainly simplify and benefit from this theory.We study the pervasive convex Euclidean bodies and their various representations.In particular, we make convex polyhedra, cones, and dual cones more visceral through illustration, andwe study the geometric relation of polyhedral cones to nonorthogonal bases biorthogonal expansion.We explain conversion between halfspace- and vertex-descriptions of convex cones,we provide formulae for determining dual cones,and we show how classic alternative systems of linear inequalities or linear matrix inequalities and optimality conditions can be explained by generalized inequalities in terms of convex cones and their duals.The conic analogue to linear independence, called conic independence, is introducedas a new tool in the study of classical cone theory; the logical next step in the progression:linear, affine, conic.Any convex optimization problem has geometric interpretation.This is a powerful attraction: the ability to visualize geometry of an optimization problem.We provide tools to make visualization easier.The concept of faces, extreme points, and extreme directions of convex Euclidean bodiesis explained here, crucial to understanding convex optimization.The convex cone of positive semidefinite matrices, in particular, is studied in depth.We mathematically interpret, for example,its inverse image under affine transformation, and we explainhow higher-rank subsets of its boundary united with its interior are convex.The Chapter on "Geometry of convex functions",observes analogies between convex sets and functions:The set of all vector-valued convex functions is a closed convex cone.Included among the examples in this chapter, we show how the real affinefunction relates to convex functions as the hyperplane relates to convex sets.Here, also, pertinent results formultidimensional convex functions are presented that are largely ignored in the literature;tricks and tips for determining their convexityand discerning their geometry, particularly with regard to matrix calculus which remains largely unsystematizedwhen compared with the traditional practice of ordinary calculus.Consequently, we collect some results of matrix differentiation in the appendices.The Euclidean distance matrix (EDM) is studied,its properties and relationship to both positive semidefinite and Gram matrices.We relate the EDM to the four classical axioms of the Euclidean metric;thereby, observing the existence of an infinity of axioms of the Euclidean metric beyondthe triangle inequality. We proceed byderiving the fifth Euclidean axiom and then explain why furthering this endeavoris inefficient because the ensuing criteria (while describing polyhedra)grow linearly in complexity and number.Some geometrical problems solvable via EDMs,EDM problems posed as convex optimization, and methods of solution arepresented;\eg, we generate a recognizable isotonic map of the United States usingonly comparative distance information (no distance information, only distance inequalities).We offer a new proof of the classic Schoenberg criterion, that determines whether a candidate matrix is an EDM. Our proofrelies on fundamental geometry; assuming, any EDM must correspond to a list of points contained in some polyhedron(possibly at its vertices) and vice versa.It is not widely known that the Schoenberg criterion implies nonnegativity of the EDM entries; proved here.We characterize the eigenvalues of an EDM matrix and then devisea polyhedral cone required for determining membership of a candidate matrix(in Cayley-Menger form) to the convex cone of Euclidean distance matrices (EDM cone); \ie,a candidate is an EDM if and only if its eigenspectrum belongs to a spectral cone for EDM^N.We will see spectral cones are not unique.In the chapter "EDM cone", we explain the geometric relationship betweenthe EDM cone, two positive semidefinite cones, and the elliptope.We illustrate geometric requirements, in particular, for projection of a candidate matrixon a positive semidefinite cone that establish its membership to the EDM cone. The faces of the EDM cone are described,but still open is the question whether all its faces are exposed as they are for the positive semidefinite cone.The classic Schoenberg criterion, relating EDM and positive semidefinite cones, isrevealed to be a discretized membership relation (a generalized inequality, a new Farkas''''''-like lemma)between the EDM cone and its ordinary dual. A matrix criterion for membership to the dual EDM cone is derived thatis simpler than the Schoenberg criterion.We derive a new concise expression for the EDM cone and its dual involvingtwo subspaces and a positive semidefinite cone."Semidefinite programming" is reviewedwith particular attention to optimality conditionsof prototypical primal and dual conic programs,their interplay, and the perturbation method of rank reduction of optimal solutions(extant but not well-known).We show how to solve a ubiquitous platonic combinatorial optimization

problem from linear algebra(the optimal Boolean solution x to Ax=b)via semidefinite program relaxation.A three-dimensional polyhedral analogue for the positive semidefinite cone of 3X3 symmetricmatrices is introduced; a tool for visualizing in 6 dimensions.In "EDM proximity"we explore methods of solution to a few fundamental and prevalentEuclidean distance matrix proximity problems; the problem of finding that Euclidean distance matrix closestto a given matrix in the Euclidean sense.We pay particular attention to the problem when compounded with rank minimization.We offer a new geometrical proof of a famous result discovered by Eckart \& Young in 1936 regarding Euclideanprojection of a point on a subset of the positive semidefinite cone comprising all positive semidefinite matriceshaving rank not exceeding a prescribed limit rho.We explain how this problem is transformed to a convex optimization for any rank rho.
In the past two decades, convex analysis and optimization have been developed in Hadamard spaces. This book represents a first attempt to give a systematic account on the subject. Hadamard spaces are complete geodesic spaces of nonpositive curvature. They include Hilbert spaces, Hadamard manifolds, Euclidean buildings and many other important spaces. While the role of Hadamard spaces in geometry and geometric group theory has been studied for a long time, first analytical results appeared as late as in the 1990s. Remarkably, it turns out that Hadamard spaces are appropriate for the theory of convex sets and convex functions outside of linear spaces. Since convexity underpins a large number of results in the geometry of Hadamard spaces, we believe that its systematic study is of substantial interest. Optimization methods then address various computational issues and provide us with approximation algorithms which may be useful in sciences and engineering. We present a detailed description of such an application to computational phylogenetics. The book is primarily aimed at both graduate students and researchers in analysis and optimization, but it is accessible to advanced undergraduate students as well.
Discrete Convex Analysis is a novel paradigm for discrete optimization that combines the ideas in continuous optimization (convex analysis) and combinatorial optimization (matroid/submodular function theory) to establish a unified theoretical framework for nonlinear discrete optimization. The study of this theory is expanding with the development of efficient algorithms and applications to a number of diverse disciplines like matrix theory, operations research, and economics. This self-contained book is designed to provide a novel insight into optimization on discrete structures and should reveal unexpected links among different disciplines. It is the first and only English-language monograph on the theory and applications of discrete convex analysis.
Convex Sets and Their ApplicationsCourier Corporation
This monograph provides an introduction to the theory of topologies defined on the closed subsets of a metric space, and on the closed convex subsets of a normed linear space as well. A unifying theme is the relationship between

topology and set convergence on the one hand, and set functionals on the other. The text includes for the first time anywhere an exposition of three topologies that over the past ten years have become fundamental tools in optimization, one-sided analysis, convex analysis, and the theory of multifunctions: the Wijsman topology, the Attouch--Wets topology, and the slice topology. Particular attention is given to topologies on lower semicontinuous functions, especially lower semicontinuous convex functions, as associated with their epigraphs. The interplay between convex duality and topology is carefully considered and a chapter on set-valued functions is included. The book contains over 350 exercises and is suitable as a graduate text. This book is of interest to those working in general topology, set-valued analysis, geometric functional analysis, optimization, convex analysis and mathematical economics.

This reference text, now in its second edition, offers a modern unifying presentation of three basic areas of nonlinear analysis: convex analysis, monotone operator theory, and the fixed point theory of nonexpansive operators. Taking a unique comprehensive approach, the theory is developed from the ground up, with the rich connections and interactions between the areas as the central focus, and it is illustrated by a large number of examples. The Hilbert space setting of the material offers a wide range of applications while avoiding the technical difficulties of general Banach spaces. The authors have also drawn upon recent advances and modern tools to simplify the proofs of key results making the book more accessible to a broader range of scholars and users. Combining a strong emphasis on applications with exceptionally lucid writing and an abundance of exercises, this text is of great value to a large audience including pure and applied mathematicians as well as researchers in engineering, data science, machine learning, physics, decision sciences, economics, and inverse problems. The second edition of Convex Analysis and Monotone Operator Theory in Hilbert Spaces greatly expands on the first edition, containing over 140 pages of new material, over 270 new results, and more than 100 new exercises. It features a new chapter on proximity operators including two sections on proximity operators of matrix functions, in addition to several new sections distributed throughout the original chapters. Many existing results have been improved, and the list of references has been updated. Heinz H. Bauschke is a Full Professor of Mathematics at the Kelowna campus of the University of British Columbia, Canada. Patrick L. Combettes, IEEE Fellow, was on the faculty of the City University of New York and of Université Pierre et Marie Curie – Paris 6 before joining North Carolina State University as a Distinguished Professor of Mathematics in 2016.

Thorough introduction to an important area of mathematics Contains recent results Includes many exercises

The importance of convexity arguments in functional analysis has long been realized, but a comprehensive theory of infinite-dimensional convex sets has hardly existed for more than a decade. In fact, the integral representation theorems of Choquet and Bishop -de Leeuw together with the uniqueness theorem of Choquet inaugurated a new epoch in infinite-dimensional convexity. Initially considered curious and tech nically difficult, these theorems attracted many mathematicians, and the proofs were gradually simplified and fitted into a general theory. The results can no longer be considered very "deep" or difficult, but they certainly remain all the more important. Today Choquet Theory provides a unified approach to integral representations in fields as diverse as potential theory, probability, function algebras, operator theory, group representations and ergodic theory. At the same time the new concepts and results have made it possible, and relevant, to ask new questions within the abstract theory itself. Such questions pertain to the interplay between compact convex sets K and their associated spaces A(K) of continuous affine functions; to the duality between faces of K and appropriate ideals of A(K); to dominated extension problems for continuous affine functions on faces; and to direct convex sum decomposition into faces, as well as to integral for mulas generalizing such decompositions. These problems are of geometric interest in their own right, but they are primarily suggested by applica tions, in particular to operator theory and function algebras. The aim of this book is to introduce the reader to the fascinating world of convex polytopes. The highlights of the book are three main theorems in the combinatorial theory of convex polytopes, known as the Dehn-Sommerville Relations, the Upper Bound Theorem and the Lower Bound Theorem. All the background information on convex sets and convex polytopes which is m~eded to under stand and appreciate these three theorems is developed in detail. This background material also forms a basis for studying other aspects of polytope theory. The Dehn-Sommerville Relations are classical, whereas the proofs of the Upper Bound Theorem and the Lower Bound Theorem are of more recent date: they were found in the early 1970's by P. McMullen and D. Barnette, respectively. A famous conjecture of P. McMullen on the charac terization off-vectors of simplicial or simple polytopes dates from the same period; the book ends with a brief discussion of this conjecture and some of its relations to the Dehn-Sommerville Relations, the Upper Bound Theorem and the Lower Bound Theorem. However, the recent proofs that McMullen's conditions are both sufficient (L. J. Billera and C. W. Lee, 1980) and necessary (R. P. Stanley, 1980) go beyond the scope of the book. Prerequisites for reading the book are modest: standard linear algebra and elementary point set topology in [R1d will suffice. Nonsmooth Analysis is a relatively recent area of mathematical analysis. The literature about this subject consists mainly in research papers and books. The purpose of this book is to provide a handbook for undergraduate and graduate students of mathematics that introduce this interesting area in detail. Includes different kinds of sub and super differentials as well as generalized gradients Includes also the main tools of the theory, as Sum and Chain Rules or Mean Value theorems Content is introduced in an elementary way, developing many examples, allowing the reader to understand a theory which is scattered in many papers and research books

This book presents state-of-the-art results and methodologies in modern global optimization, and has been a staple reference for researchers, engineers, advanced students (also in applied mathematics), and practitioners in various fields of engineering. The second edition has been brought up to date and continues to develop a coherent and rigorous theory of deterministic global optimization, highlighting the essential role of convex analysis. The text has been revised and expanded to meet the needs of research, education, and applications for many years to come. Updates for this new edition include: · Discussion of modern approaches to minimax, fixed point, and equilibrium theorems, and to nonconvex optimization; · Increased focus on dealing more efficiently with ill-posed problems of global optimization, particularly those with hard constraints; · Important discussions of decomposition methods for specially structured problems; · A complete revision of the chapter on nonconvex quadratic programming, in order to encompass the advances made in quadratic optimization since publication of the first edition. · Additionally, this new edition contains entirely new chapters devoted to monotonic optimization, polynomial optimization and optimization under equilibrium constraints, including bilevel programming, multiobjective programming, and optimization with variational inequality constraint. From the reviews of the first edition: The book gives a good review of the topic. ...The text is carefully constructed and well written, the exposition is clear. It leaves a remarkable impression of the concepts, tools and techniques in global optimization. It might also be used as a basis and guideline for lectures on this subject. Students as well as professionals will profitably read and use it.—Mathematical Methods of Operations Research, 49:3 (1999)

This book focuses on the problem of linear approximability, or decomposability for distribution functions. While questions concerning this topics had been raised long ago, only ?ad hoc? procedures had been found out. Here, the author deals with the treatment of general methods for this problem.

A gentle introduction to the geometry of convex sets in n–dimensional space Geometry of Convex Sets begins with basic definitions of the linear concepts of addition and scalar multiplication and then defines the notion of convexity for subsets of n–dimensional space. Many properties of convex sets can be discovered using just the linear structure. However, for more interesting results, it is necessary to discuss the notion of distance about open sets, closed sets, bounded sets, and compact sets. The book illustrates the interplay between these linear and topological concepts, which makes the notion of convexity so appealing. Thoroughly class–tested, the book discusses topology and convexity in the context of normed linear spaces, specifically with a norm topology on an n–dimensional space. Geometry of Convex Sets also features: An introduction to n–dimensional geometry including points; lines; vectors; distance; norms; inner products; orthogonality; convexity; hyperplanes; and linear functionals An introduction to n–dimensional norm topology including interior points and open sets; accumulation points and closed sets; boundary points and closed sets; compact subsets of n–dimensional space; completeness of n–dimensional space;

sequences; equivalent norms; distance between sets; and support hyperplanes Basic properties of convex sets; convex hulls; interior and closure of convex sets; closed convex hulls; accessibility lemma; regularity of convex sets; affine hulls; flats or affine subspaces; affine basis theorem; separation theorems; extreme points of convex sets; supporting hyperplanes and extreme points; existence of extreme points; Krein–Milman theorem; polyhedral sets and polytopes; and Birkhoff s theorem on doubly stochastic matrices Discussions on Helly s theorem; the Art Gallery theorem; Vincensini s problem; Hadwiger s theorems; theorems of Radon and Caratheodory; Kirchberger s theorem; Helly–type theorems for circles; covering problems; piercing problems; sets of constant width; Reuleaux triangles; Barbier s theorem; and Borsuk s problem Geometry of Convex Sets is a useful textbook for upper–undergraduate level courses in geometry of convex sets and is essential for graduate level courses in convex analysis. An excellent reference for academics and readers interested in learning the various applications of higher geometry, the book is also appropriate for teachers who would like to convey a better understanding and appreciation of the field to students.
Many important applications in global optimization, algebra, probability and statistics, applied mathematics, control theory, financial mathematics, inverse problems, etc. can be modeled as a particular instance of the Generalized Moment Problem (GMP) . This book introduces a new general methodology to solve the GMP when its data are polynomials and basic semi-algebraic sets. This methodology combines semidefinite programming with recent results from real algebraic geometry to provide a hierarchy of semidefinite relaxations converging to the desired optimal value. Applied on appropriate cones, standard duality in convex optimization nicely expresses the duality between moments and positive polynomials. In the second part, the methodology is particularized and described in detail for various applications, including global optimization, probability, optimal control, mathematical finance, multivariate integration, etc., and examples are provided for each particular application. Errata(s). Errata. Sample Chapter(s). Chapter 1: The Generalized Moment Problem (227 KB). Contents: Moments and Positive Polynomials: The Generalized Moment Problem; Positive Polynomials; Moments; Algorithms for Moment Problems; Applications: Global Optimization over Polynomials; Systems of Polynomial Equations; Applications in Probability; Markov Chains Applications; Application in Mathematical Finance; Application in Control; Convex Envelope and Representation of Convex Sets; Multivariate Integration; Min-Max Problems and Nash Equilibria; Bounds on Linear PDE. Readership: Postgraduates, academics and researchers in mathematical programming, control and optimization.
Suitable for advanced undergraduates and graduate students, this text introduces the broad scope of convexity. It leads students to open questions and unsolved problems, and it highlights diverse applications. Author Steven R. Lay, Professor of Mathematics at Lee University in Tennessee, reinforces his teachings with numerous examples, plus exercises with hints and answers. The first three chapters form the foundation for all that follows, starting with a review of the fundamentals of linear algebra and topology. They also survey the development and applications of relationships between hyperplanes and convex sets. Subsequent chapters are relatively self-contained, each focusing on a particular aspect or application of convex sets. Topics include characterizations of convex sets, polytopes, duality, optimization, and convex functions.

Hints, solutions, and references for the exercises appear at the back of the book.
Handbook of Convex Geometry, Volume A offers a survey of convex geometry and its many ramifications and relations with other areas of mathematics, including convexity, geometric inequalities, and convex sets. The selection first offers information on the history of convexity, characterizations of convex sets, and mixed volumes. Topics include elementary convexity, equality in the Aleksandrov-Fenchel inequality, mixed surface area measures, characteristic properties of convex sets in analysis and differential geometry, and extensions of the notion of a convex set. The text then reviews the standard isoperimetric theorem and stability of geometric inequalities. The manuscript takes a look at selected affine isoperimetric inequalities, extremum problems for convex discs and polyhedra, and rigidity. Discussions focus on include infinitesimal and static rigidity related to surfaces, isoperimetric problem for convex polyhedral, bounds for the volume of a convex polyhedron, curvature image inequality, Busemann intersection inequality and its relatives, and Petty projection inequality. The book then tackles geometric algorithms, convexity and discrete optimization, mathematical programming and convex geometry, and the combinatorial aspects of convex polytopes. The selection is a valuable source of data for mathematicians and researchers interested in convex geometry.
The book is devoted to the theory of pairs of compact convex sets and in particular to the problem of finding different types of minimal representants of a pair of nonempty compact convex subsets of a locally convex vector space in the sense of the Rådström-Hörmander Theory. Minimal pairs of compact convex sets arise naturally in different fields of mathematics, as for instance in non-smooth analysis, set-valued analysis and in the field of combinatorial convexity. In the first three chapters of the book the basic facts about convexity, mixed volumes and the Rådström-Hörmander lattice are presented. Then, a comprehensive theory on inclusion-minimal representants of pairs of compact convex sets is given. Special attention is given to the two-dimensional case, where the minimal pairs are uniquely determined up to translations. This fact is not true in higher dimensional spaces and leads to a beautiful theory on the mutual interactions between minimality under constraints, separation and decomposition of convex sets, convexificators and invariants of minimal pairs. This theory throws light upon both sides of the collection of all compact convex subsets of a locally vector space, namely the geometric and the algebraic one. From the algebraic point of view the collection of all nonempty compact convex subsets of a topological vector space is an ordered semi group with cancellation property under the inclusion of sets and the Minkowski-addition. From this approach pairs of nonempty compact convex sets correspond to fractions of elements from the semi group and minimal pairs to relatively prime fractions.
Convex Optimization for Signal Processing and Communications: From Fundamentals to Applications provides fundamental background knowledge of convex optimization, while striking a balance between mathematical theory and applications in signal processing and communications. In addition to comprehensive proofs and perspective interpretations for core convex optimization theory, this book also provides many insightful figures, remarks, illustrative examples, and guided journeys from theory to cutting-edge research explorations, for efficient and in-depth learning, especially for engineering students and professionals. With the powerful convex optimization theory and tools, this book provides you with a new degree of freedom and the capability of

solving challenging real-world scientific and engineering problems.

In the last few years, Algorithms for Convex Optimization have revolutionized algorithm design, both for discrete and continuous optimization problems. For problems like maximum flow, maximum matching, and submodular function minimization, the fastest algorithms involve essential methods such as gradient descent, mirror descent, interior point methods, and ellipsoid methods. The goal of this self-contained book is to enable researchers and professionals in computer science, data science, and machine learning to gain an in-depth understanding of these algorithms. The text emphasizes how to derive key algorithms for convex optimization from first principles and how to establish precise running time bounds. This modern text explains the success of these algorithms in problems of discrete optimization, as well as how these methods have significantly pushed the state of the art of convex optimization itself.

This volume is dedicated to the fundamentals of convex functional analysis. It presents those aspects of functional analysis that are extensively used in various applications to mechanics and control theory. The purpose of the text is essentially two-fold. On the one hand, a bare minimum of the theory required to understand the principles of functional, convex and set-valued analysis is presented. Numerous examples and diagrams provide as intuitive an explanation of the principles as possible. On the other hand, the volume is largely self-contained. Those with a background in graduate mathematics will find a concise summary of all main definitions and theorems.

The book provides a self-contained and systematic treatment of algebraic and topological properties of convex sets in the n-dimensional Euclidean space. It benefits advanced undergraduate and graduate students with various majors in mathematics, optimization, and operations research. It may be adapted as a primary book or an additional text for any course in convex geometry or convex analysis, aimed at non-geometers. It can be a source for independent study and a reference book for researchers in academia.The second edition essentially extends and revises the original book. Every chapter is rewritten, with many new theorems, examples, problems, and bibliographical references included. It contains three new chapters and 100 additional problems with solutions.

The purpose of this volume is to give an up-to-date introduction to tensor valuations and their applications. Starting with classical results concerning scalar-valued valuations on the families of convex bodies and convex polytopes, it proceeds to the modern theory of tensor valuations. Product and Fourier-type transforms are introduced and various integral formulae are derived. New and well-known results are presented, together with generalizations in several directions, including extensions to the non-Euclidean setting and to non-convex sets. A variety of applications of tensor valuations to models in stochastic geometry, to local stereology and to imaging are also discussed.

A comprehensive textbook for advanced undergraduate or graduate students.

A comprehensive introduction to the tools, techniques and applications of convex optimization.

Lectori salutem! The kind reader opens the book that its authors would have liked to read it themselves, but it was not written yet. Then, their only choice was to write this book, to fill a gap in the mathematicalliterature. The idea of convexity has appeared in the human mind since the antiquity and its fertility has led to a huge diversity of notions and of applications. A student intending a thoroughgoing study of convexity has the sensation of swimming into an ocean. It is due to two reasons: the first one is the great number of properties and applications of the classical convexity and second one is the great number of generalisations for various purposes. As a consequence, a tendency of writing huge books guiding the reader in convexity appeared during the last twenty years (for example, the books of P. M. Gruber and J. M. Willis (1993) and R. J. Webster (1994)). Another last years' tendency is to order, from some point of view, as many convexity notions as possible (for example, the book of I. Singer (1997)). These approaches to the domain of convexity follow the previous point of view of axiomatizing it (A. Ghika (1955), W. Prenowitz (1961), D. Voiculescu (1967), V. W. Bryant and R. J. Webster (1969)). Following this last tendency, our book proposes to the reader two classifications of convexity properties for sets, both of them starting from the internal mechanism of defining them.

An accessible introduction to convex algebraic geometry and semidefinite optimization. For graduate students and researchers in mathematics and computer science.

A Solutions Manual to accompany Geometry of Convex Sets Geometry of Convex Sets begins with basic definitions of the concepts of vector addition and scalar multiplication and then defines the notion of convexity for subsets of n-dimensional space. Many properties of convex sets can be discovered using just the linear structure. However, for more interesting results, it is necessary to introduce the notion of distance in order to discuss open sets, closed sets, bounded sets, and compact sets. The book illustrates the interplay between these linear and topological concepts, which makes the notion of convexity so interesting. Thoroughly class-tested, the book discusses topology and convexity in the context of normed linear spaces, specifically with a norm topology on an n-dimensional space. Geometry of Convex Sets also features: An introduction to n-dimensional geometry including points; lines; vectors; distance; norms; inner products; orthogonality; convexity; hyperplanes; and linear functionals Coverage of n-dimensional norm topology including interior points and open sets; accumulation points and closed sets; boundary points and closed sets; compact subsets of n-dimensional space; completeness of n-dimensional space; sequences; equivalent norms; distance between sets; and support hyperplanes · Basic properties of convex sets; convex hulls; interior and closure of convex sets; closed convex hulls; accessibility lemma; regularity of convex sets; affine hulls; flats or affine subspaces; affine basis theorem; separation theorems; extreme points of convex sets; supporting hyperplanes and extreme points; existence of extreme points; Krein–Milman theorem; polyhedral sets and polytopes; and Birkhoff's theorem on doubly stochastic matrices Discussions of Helly's theorem; the Art Gallery theorem; Vincensini's problem; Hadwiger's theorems; theorems of Radon and Caratheodory; Kirchberger's theorem; Helly-type theorems for circles; covering problems; piercing problems; sets of constant width; Reuleaux triangles; Barbier's theorem; and Borsuk's problem Geometry of Convex Sets is a useful textbook for upper-undergraduate level courses in geometry of convex sets and is essential for graduate-level courses in convex analysis. An excellent reference for academics and readers interested in learning the various applications of convex geometry, the book is also appropriate for teachers who would like to convey a better understanding and appreciation of the field to students. I. E. Leonard, PhD, was a contract lecturer in the Department of Mathematical and Statistical Sciences at the University of Alberta. The author of over 15 peer-reviewed journal articles, he is a technical editor for the Canadian Applied Mathematical Quarterly journal. J. E. Lewis, PhD, is Professor Emeritus in the Department of Mathematical Sciences at the University of Alberta. He was the recipient of the Faculty of Science Award for Excellence in Teaching in 2004 as well as the PIMS Education Prize in 2002.

This tutorial is the first comprehensive introduction to (possibly infinite) linear systems containing strict inequalities and evenly convex sets. The book introduces their application to convex optimization. Particular attention is paid to evenly convex polyhedra and finite linear systems containing strict inequalities. The book also analyzes evenly convex and quasiconvex functions from a conjugacy and duality perspective. It discusses the applications of these functions in economics. Written in an expository style the main concepts and basic results are illustrated with suitable examples and figures..

This account of convexity includes the basic properties of convex sets in Euclidean space and their applications, the theory of convex functions and an outline of the results of transformations and combinations of convex sets. It will be useful for those concerned with the many applications of convexity in economics, the theory of games, the theory of functions, topology, geometry and the theory of numbers.

A uniquely pedagogical, insightful, and rigorous treatment of the analytical/geometrical foundations of optimization. The book provides a comprehensive development of convexity theory, and its rich applications in optimization, including duality, minimax/saddle point theory, Lagrange multipliers, and Lagrangian relaxation/nondifferentiable optimization. It is an excellent supplement to several of our books: Convex Optimization Theory (Athena Scientific, 2009), Convex Optimization Algorithms (Athena Scientific, 2015), Nonlinear Programming (Athena Scientific, 2016), Network Optimization (Athena Scientific, 1998), and Introduction to Linear Optimization (Athena Scientific, 1997). Aside from a thorough account of convex analysis and optimization, the book aims to restructure the theory of the subject, by introducing several novel unifying lines of analysis, including: 1) A unified development of minimax theory and constrained optimization duality as special cases of duality between two simple geometrical problems. 2) A unified development of conditions for existence of solutions of convex optimization problems, conditions for the minimax equality to hold, and conditions for the absence of a duality gap in constrained optimization. 3) A unification of the major constraint qualifications allowing the use of Lagrange multipliers for nonconvex constrained optimization, using the notion of constraint pseudonormality and an enhanced form of the Fritz John necessary optimality conditions. Among its features the book: a) Develops rigorously and comprehensively the theory of convex sets and functions, in the classical tradition of Fenchel and Rockafellar b) Provides a geometric, highly visual treatment of convex and nonconvex optimization problems, including existence of solutions, optimality conditions, Lagrange multipliers, and duality c) Includes an insightful and comprehensive presentation of minimax theory and zero sum games, and its connection with duality d) Describes dual optimization, the associated computational methods, including the novel incremental subgradient methods, and applications in linear, quadratic, and integer programming e) Contains many examples, illustrations, and exercises with complete solutions (about 200 pages) posted at the publisher's web site http://www.athenasc.com/convexity.html