Category Theory Lecture Notes University Of Edinburgh

A Collection of Lectures by Variuos Authors

Basic Category Theory for Computer Scientists provides a straightforward presentation of the basic constructions and terminology of category theory, including limits, functors, natural transformations, adjoints, and cartesian closed categories. Category theory is a branch of pure mathematics that is becoming an increasingly important tool in theoretical computer science, especially in programming language semantics, domain theory, and concurrency, where it is already a standard language of discourse. Assuming a minimum of mathematical preparation, Basic Category Theory for Computer Scientists provides a straightforward presentation of the basic constructions and terminology of category theory, including limits, functors, natural transformations, adjoints, and cartesian closed categories. Four case studies illustrate applications of category theory to programming language design, semantics, and the solution of recursive domain equations. A brief literature survey offers suggestions for further study in more advanced texts. Contents Tutorial • Applications • Further Reading

This volume explores the many different meanings of the notion of the axiomatic method, offering an insightful historical and philosophical discussion about how these notions changed over the millennia. The author, a well-known philosopher and historian of mathematics, first examines Euclid, who is considered the father of the axiomatic method, before moving onto Hilbert and Lawvere. He then presents a deep textual analysis of each writer and describes how their ideas are different and even how their ideas progressed over time. Next, the book explores category theory and details how it has revolutionized the notion of the axiomatic method. It considers the question of identity/equality in mathematics as well as examines the received theories of mathematical structuralism. In the end, Rodin presents a hypothetical New Axiomatic Method, which establishes closer relationships between mathematics and physics. Lawvere's axiomatization of topos theory and Voevodsky's axiomatization of higher homotopy theory exemplify a new way of axiomatic theory building, which goes beyond the classical Hilbert-style Axiomatic Method. The new notion of Axiomatic Method that emerges in categorical logic opens new possibilities for using this method in physics and other natural sciences. This volume offers readers a coherent look at the past, present and anticipated future of the Axiomatic Method.

Mathematicians interested in understanding the directions of current research in set theory will not want to overlook this book, which contains the proceedings of the AMS Summer Research Conference on Axiomatic Set Theory, held in Boulder, Colorado, June 19-25, 1983. This was the first large meeting devoted exclusively to set theory since the legendary 1967 UCLA meeting, and a large majority of the most active research mathematicians in the field participated. All areas of set theory, including constructibility, forcing, combinatorics and descriptive set theory, were represented; many of the papers in the proceedings explore connections between areas. Readers should have a background of graduate-level set theory. There is a paper by S. Shelah applying proper forcing to obtain consistency results on combinatorial cardinal 'invariants' below the continuum, and papers by R.

David and S. Freidman on properties of $0^\ No. $.Papers by A. Blass, H.D. Donder, T. Jech and W. Mitchell involve inner models with measurable cardinals and various combinatorial properties. T. Carlson largely solves the pin-up problem, and D. Velleman presents a novel construction of a Souslin tree from a morass. S. Todorcevic obtains the strong failure of the \qedprinciple from the Proper Forcing Axiom and A. Miller discusses properties of a new species of perfect-set forcing. H. Becker and A. Kechris attack the third Victoria Delfino problem while W. Zwicker looks at combinatorics on $P_\kappa(\lambda)$ and J. Henle studies infinite-exponent partition relations. A. Blass shows that if every vector space has a basis then AC holds. I. Anellis treats the history of set theory, and W. Fleissner presents set-theoretical axioms of use in general topology.

This book constitutes the refereed proceedings of the 7th International Conference on Category Theory and Computer Science, CTCS'97, held in Santa Margheria Ligure, Italy, in September 1997. Category theory attracts interest in the theoretical computer science community because of its ability to establish connections between different areas in computer science and mathematics and to provide a few generic principles for organizing mathematical theories. This book presents a selection of 15 revised full papers together with three invited contributions. The topics addressed include reasoning principles for types, rewriting, program semantics, and structuring of logical systems.

A wide coverage of topics in category theory and computer science is developed in this text, including introductory treatments of cartesian closed categories, sketches and elementary categorical model theory, and triples. Over 300 exercises are included.

This book has a fundamental relationship to the International Seminar on Fuzzy Set Theory held each September in Linz, Austria. First, this volume is an extended account of the eleventh Seminar of 1989. Second, and more importantly, it is the culmination of the tradition of the preceding ten Seminars. The purpose of the Linz Seminar, since its inception, was and is to foster the development of the mathematical aspects of fuzzy sets. In the earlier years, this was accomplished by bringing together for a week small grou ps of mathematicians in various fields in an intimate, focused environment which promoted much informal, critical discussion in addition to formal presentations. Beginning with the tenth Seminar, the intimate setting was retained, but each Seminar narrowed in theme; and participation was broadened to include both younger scholars within, and established mathematicians outside, the mathematical mainstream of fuzzy sets theory. Most of the material of this book was developed over the years in close association with the Seminar or influenced by what transpired at Linz. For much of the content, it played a crucial role in either stimulating this material or in providing feedback and the necessary screening of ideas. Thus we may fairly say that the book, and the eleventh Seminar to which it is directly related, are in many respects a culmination of the previous Seminars.

This monograph introduces involutive categories and involutive operads, featuring applications to the GNS construction and algebraic quantum field theory. The author adopts an accessible approach for readers seeking an overview of involutive category theory, from the basics to cutting-edge applications. Additionally, the author's own recent advances in the area are featured, never having appeared previously in the literature. The opening chapters offer an introduction to basic category theory, ideal for readers new to the area. Chapters three through five feature previously unpublished results on coherence and strictification of involutive categories and involutive monoidal categories, showcasing the author's state-of-the-art research. Chapters on coherence of involutive symmetric monoidal categories, and categorical GNS construction follow. The last chapter covers involutive operads and lays important coherence foundations for applications to algebraic quantum field theory. With detailed explanations and exercises throughout, Involutive Category Theory is suitable for graduate seminars and independent study. Mathematicians and mathematical physicists who use involutive objects will also find this a valuable reference.

Basic Category TheoryCambridge University Press

As category theory approaches its first half-century, it continues to grow, finding new applications in areas that would have seemed inconceivable a generation ago, as well as in more traditional areas. The language, ideas, and techniques of category theory are well suited to discovering unifying structures in apparently different contexts. Representing this diversity of the field, this book contains the proceedings of an international conference on category theory. The subjects covered here range from topology and geometry to logic and theoretical computer science, from homotopy to braids and conformal field theory. Although generally aimed at experts in the various fields represented, the book will also provide an excellent opportunity for nonexperts to get a feel for the diversity of current applications of category theory.

Category theory is unmatched in its ability to organize and layer abstractions and to find commonalities between structures of all sorts. No longer the exclusive preserve of pure mathematicians, it is now proving itself to be a powerful tool in science, informatics, and industry. By facilitating communication between communities and building rigorous bridges between disparate worlds, applied category theory has the potential to be a major organizing force. This book offers a self-contained tour of applied category theory. Each chapter follows a single thread motivated by a real-world application and discussed with category-theoretic tools. We see data migration as an adjoint functor, electrical circuits in terms of monoidal categories and operads, and collaborative design via enriched profunctors. All the relevant category theory, from simple to sophisticated, is introduced in an accessible way with many examples and exercises, making this an ideal guide even for those without experience of university-level mathematics.

This book gives an axiomatic presentation of stable homotopy theory. It starts with axioms defining a 'stable homotopy category'; using these axioms, one can make various constructions - cellular towers, Bousfield localization, and Brown representability, to name a few. Much of the book is devoted to these constructions and to the study of the global structure of stable homotopy categories. Next, a number of examples of such categories are presented. Some of these arise in topology (the ordinary stable homotopy category of spectra, categories of equivariant spectra, and Bousfield localizations of these), and others in algebra (coming from the representation theory of groups or of Lie algebras, as well as the derived category of a commutative ring). Hence one can apply many of the tools of stable homotopy theory to these algebraic situations.This work: provides a reference for standard results and constructions in stable homotopy theory; discusses applications of those results to algebraic settings, such as group theory and commutative algebra; provides a unified treatment of several different situations in stable homotopy, including equivariant stable homotopy and localizations of the stable homotopy category; and, also provides a context for nilpotence and thick subcategory theorems, such as the nilpotence theorem of Devinatz-Hopkins-Smith and the thick subcategory theorem of Hopkins-Smith in stable homotopy theory, and the thick subcategory theorem of Benson-Carlson-Rickard in representation theory. This book presents stable homotopy theory as a branch of mathematics in its own right with applications in other fields of mathematics. It is a first step toward making stable homotopy theory a tool useful in many disciplines of mathematics.

This categorical perspective on homotopy theory helps consolidate and simplify one's understanding of derived functors, homotopy limits and colimits, and model categories, among others.

Category Theory has developed rapidly. This book aims to present those ideas and methods which can now be effectively used by Mathe maticians working in a variety of other fields of Mathematical research. This occurs at several levels. On the first level, categories provide a convenient conceptual language, based on the notions of category, functor, natural transformation, contravariance, and functor category. These notions are presented, with appropriate examples, in Chapters I and II. Next comes the fundamental idea of an adjoint pair of functors. This appears in many substantially equivalent forms: That of universal construction, that of direct and inverse limit, and that of pairs offunctors with a natural isomorphism between corresponding sets of arrows. All these forms, with their interrelations, are examined in Chapters III to V. The slogan is "Adjoint functors arise everywhere". Alternatively, the fundamental notion of category theory is that of a monoid -a set with a binary operation of multiplication which is associative and which has a unit; a category itself can be regarded as a sort of general ized monoid. Chapters VI and VII explore this notion and its generaliza tions. Its close connection to pairs of adjoint functors illuminates the ideas of universal algebra and culminates in Beck's theorem characterizing categories of algebras; on the other hand, categories with a monoidal structure (given by a tensor product) lead inter alia to the study of more convenient categories of topological spaces.

An array of general ideas useful in a wide variety of fields. Starting from the foundations, this book illuminates the concepts of category, functor, natural transformation, and duality. It then turns to adjoint functors, which provide a description of universal constructions, an analysis of the representations of functors by sets of morphisms, and a means of manipulating direct and inverse limits. These categorical concepts are extensively illustrated in the remaining chapters, which include many applications of the basic existence theorem for adjoint functors. The categories of algebraic systems are constructed from certain adjoint-like data and characterised by Beck's theorem. After considering a variety of applications, the book continues with the construction and exploitation of Kan extensions. This second edition includes a number of revisions and additions, including new chapters on topics of active interest: symmetric monoidal categories and braided monoidal categories, and the coherence theorems for them, as well as 2-categories and the higher dimensional categories which have recently come into prominence.

This book develops abstract homotopy theory from the categorical perspective with a particular focus on examples. Part I discusses two competing perspectives by which one typically first encounters homotopy (co)limits: either as derived functors definable when the appropriate diagram categories admit a compatible model structure, or through particular formulae that give the right notion in certain examples. Emily Riehl unifies these seemingly rival perspectives and demonstrates that model structures on diagram categories are irrelevant. Homotopy (co)limits are explained to be a special case of weighted (co)limits, a foundational topic in enriched category theory. In Part II, Riehl further examines this topic, separating categorical arguments from homotopical ones. Part III treats the most ubiquitous axiomatic framework for homotopy theory - Quillen's model categories. Here, Riehl simplifies familiar model categorical lemmas and definitions by focusing on weak factorization systems. Part IV introduces quasi-categories and homotopy coherence.

An introduction to category theory as a rigorous, flexible, and coherent modeling language that can be used across the sciences.

Category theory was invented in the 1940s to unify and synthesize different areas in mathematics, and it has proven remarkably successful in enabling powerful communication between disparate fields and subfields within mathematics. This book shows that category theory can be useful outside of mathematics as a rigorous, flexible, and coherent modeling language throughout the sciences. Information is inherently dynamic; the same ideas can be organized and reorganized in countless ways, and the ability to translate between such organizational structures is becoming increasingly important in the sciences. Category theory offers a unifying framework for information modeling that can facilitate the translation of knowledge between disciplines. Written in an engaging and straightforward style, and assuming little background in mathematics, the book is rigorous but accessible to non-mathematicians. Using databases as an entry to category theory, it begins with sets and functions, then introduces the reader to notions that are fundamental in mathematics: monoids, groups, orders, and graphs—categories in disguise. After explaining the "big three" concepts of category theory—categories, functors, and natural transformations—the book covers other topics, including limits, colimits, functor categories, sheaves, monads, and operads. The book explains category theory by examples and exercises rather than focusing on theorems and proofs. It includes more than 300 exercises, with solutions. Category Theory for the Sciences is intended to create a bridge between the vast array of mathematical concepts used by mathematicians and the models and frameworks of such scientific disciplines as computation, neuroscience, and physics.

Bridge the gap between category theory and its applications in homotopy theory with this guide for graduate students and researchers.

A short introduction ideal for students learning category theory for the first time.

Category Theory now permeates most of Mathematics, large parts of theoretical Computer Science and parts of theoretical Physics. Its unifying power brings together different branches, and leads to a better understanding of their roots.This book is addressed to students and researchers of these fields and can be used as a text for a first course in Category Theory. It covers the basic tools, like universal properties, limits, adjoint functors and monads. These are presented in a concrete way, starting from examples and exercises taken from elementary Algebra, Lattice Theory and Topology, then developing the theory together with new exercises and applications.A reader should have some elementary knowledge of these three subjects, or at least two of them, in order to be able to follow the main examples, appreciate the unifying power of the categorical approach, and discover the subterranean links brought to light and formalised by this perspective.Applications of Category Theory form a vast and differentiated domain. This book wants to present the basic applications in Algebra and Topology, with a choice of more advanced ones, based on the interests of the author. References are given for applications in many other fields.In this second edition, the book has been entirely reviewed, adding many applications and exercises. All non-obvious exercises have now a solution (or a reference, in the case of an advanced topic); solutions are now collected in the last chapter.

The book "Foundational Theories of Classical and Constructive Mathematics" is a book on the classical topic of foundations of mathematics. Its originality resides mainly in its treating at the same time foundations of classical and foundations of constructive mathematics. This confrontation of two kinds of foundations contributes to answering questions such as: Are foundations/foundational theories of classical mathematics of a different nature compared to those of constructive mathematics? Do they play the same role for the resp. mathematics? Are there connections between the two kinds of foundational theories? etc. The confrontation and comparison is often implicit and sometimes explicit. Its great advantage is to extend the traditional discussion of the foundations of mathematics and to render it at the same time more subtle and more differentiated. Another important aspect of the book is that some of its contributions are of a more philosophical, others of a more technical nature. This double face is emphasized, since foundations of mathematics is an eminent topic in the philosophy of mathematics: hence both sides of this discipline ought to be and are being paid due to.

Category theory has become the universal language of modern mathematics. This book is a collection of articles applying methods of category theory to the areas of algebra, geometry, and mathematical physics. Among others, this book contains articles on higher categories and their applications and on homotopy theoretic methods. The reader can learn about the exciting new interactions of category theory with very traditional mathematical disciplines.

With one exception, these papers are original and fully refereed research articles on various applications of Category Theory to Algebraic Topology, Logic and Computer Science. The exception is an outstanding and lengthy survey paper by Joyal/Street (80 pp) on a growing subject: it gives an account of classical Tannaka duality in such a way as to be accessible to the general mathematical reader, and to provide a key for entry to more recent developments and quantum groups. No expertise in either representation theory or category theory is assumed. Topics such as the Fourier cotransform, Tannaka duality for homogeneous spaces, braided tensor categories, Yang-Baxter operators, Knot invariants and quantum groups are introduced and studies. From the Contents: P.J. Freyd: Algebraically complete categories.- J.M.E. Hyland: First steps in synthetic domain theory.- G. Janelidze, W. Tholen: How algebraic is the change-of-base functor?.- A. Joyal, R. Street: An introduction to Tannaka duality and quantum groups.- A. Joyal, M. Tierney: Strong stacks andclassifying spaces.- A. Kock: Algebras for the partial map classifier monad.- F.W. Lawvere: Intrinsic co-Heyting boundaries and the Leibniz rule in certain toposes.- S.H. Schanuel: Negative sets have Euler characteristic and dimension.-

The contributions gathered here demonstrate how categorical ontology can provide a basis for linking three important basic sciences: mathematics, physics, and philosophy. Category theory is a new formal ontology that shifts the main focus from objects to processes. The book approaches formal ontology in the original sense put forward by the philosopher Edmund Husserl, namely as a science that deals with entities that can be exemplified in all spheres and domains of reality. It is a dynamic, processual, and non-substantial ontology in which all entities can be treated as transformations, and in which objects are merely the sources and aims of these transformations. Thus, in a rather surprising way, when employed as a formal ontology, category theory can unite seemingly disparate disciplines in contemporary science and the humanities, such as physics, mathematics and philosophy, but also computer and complex systems science.

The notion of a motive is an elusive one, like its namesake "the motif" of Cezanne's impressionist method of painting. Its existence was first suggested by Grothendieck in 1964 as the underlying structure behind the myriad cohomology theories in Algebraic Geometry. We now know that there is a triangulated theory of motives, discovered by Vladimir Voevodsky, which suffices for the development of a satisfactory Motivic Cohomology theory. However, the existence of motives themselves remains conjectural. This book provides an account of the triangulated theory of motives. Its purpose is to introduce Motivic Cohomology, to develop its main properties, and finally to relate it to other known invariants of algebraic varieties and rings such as Milnor K-theory, etale cohomology, and Chow groups. The book is divided into lectures, grouped in six parts. The first part presents the definition of Motivic Cohomology, based upon the notion of presheaves with transfers. Some elementary comparison theorems are given in this part. The theory of (etale, Nisnevich, and Zariski) sheaves with transfers is developed in parts two, three, and six, respectively. The theoretical core of the book is the fourth part, presenting the triangulated category of motives. Finally, the comparison with higher Chow groups is developed in part five. The lecture notes format is designed for the book to be read by an advanced graduate student or an expert in a related field. The lectures roughly correspond to one-hour lectures given by Voevodsky during the course he gave at the Institute for Advanced Study in Princeton on this subject in 1999-2000. In addition, many of the original proofs have been simplified and improved so that this book will also be a useful tool for research mathematicians. Information for our distributors: Titles in this series are copublished with the Clay Mathematics Institute (Cambridge, MA).

Monoidal category theory serves as a powerful framework for describing logical aspects of quantum theory, giving an abstract language for parallel and sequential composition, and a conceptual way to understand many high-level quantum phenomena. This text lays the foundation for this categorical quantum mechanics, with an emphasis on the graphical calculus which makes computation intuitive. Biproducts and dual objects are introduced and used to model superposition and entanglement, with quantum teleportation studied abstractly using these structures. Monoids, Frobenius structures and Hopf algebras are described, and it is shown how they can be used to model classical information and complementary observables. The CP construction, a categorical tool to describe probabilistic quantum systems, is also investigated. The last chapter introduces higher categories, surface diagrams and 2-Hilbert spaces, and shows how the language of duality in monoidal 2-categories can be used to reason about quantum protocols, including quantum teleportation and dense coding. Prior knowledge of linear algebra, quantum information or category theory would give an ideal background for studying this text, but it is not assumed, with essential background material given in a self-contained introductory chapter. Throughout the text links with many other areas are highlighted, such as representation theory, topology, quantum algebra, knot theory, and probability theory, and nonstandard models are presented, such as sets and relations. All results are stated rigorously, and full proofs are given as far as possible, making this book an invaluable reference for modern techniques in quantum logic, with much of the material not available in any other textbook.

A comprehensive reference to category theory for students and researchers in mathematics, computer science, logic, cognitive science, linguistics, and philosophy. Useful for self-study and as a course text, the book includes all basic definitions and theorems (with full proofs), as well as numerous examples and exercises.

This volume provides a series of tutorials on mathematical structures which recently have gained prominence in physics, ranging from quantum foundations, via quantum information, to quantum gravity. These include the theory of monoidal categories and corresponding graphical calculi, Girard’s linear logic, Scott domains, lambda calculus and corresponding logics for typing, topos theory, and more general process structures. Most of these structures are very prominent in computer science; the chapters here are tailored towards an audience of physicists.

Part of a four-volume set, this book constitutes the refereed proceedings of the 7th International Conference on Computational Science, ICCS 2007, held in Beijing, China in May 2007. The papers cover a large volume of topics in computational science and related areas, from multiscale physics to wireless networks, and from graph theory to tools for program development.

Conceptual progress in fundamental theoretical physics is linked with the search for the suitable mathematical structures that model the physical systems. Quantum field theory (QFT) has proven to be a rich source of ideas for mathematics for a long time. However, fundamental questions such as ``What is a QFT?'' did not have satisfactory mathematical answers, especially on spaces with arbitrary topology, fundamental for the formulation of perturbative string theory. This book contains a collection of papers highlighting the mathematical foundations of QFT and its relevance to perturbative string theory as well as the deep techniques that have been emerging in the last few years. The papers are organized under three main chapters: Foundations for Quantum Field Theory, Quantization of Field Theories, and Two-Dimensional Quantum Field Theories. An introduction, written by the editors, provides an overview of the main underlying themes that bind together the papers in the volume.

The papers in this volume were presented at the fourth biennial Summer Conference on Category Theory and Computer Science, held in Paris, September3-6, 1991. Category theory continues to be an important tool in foundationalstudies in computer science. It has been widely applied by logicians to get concise interpretations of many logical concepts. Links between logic and computer science have been developed now for over twenty years, notably via the Curry-Howard isomorphism which identifies programs with proofs and types with propositions. The triangle category theory - logic - programming presents a rich world of interconnections. Topics covered in this volume include the following. Type theory: stratification of types and propositions can be discussed in a categorical setting. Domain theory: synthetic domain theory develops domain theory internally in the constructive universe of the effective topos. Linear logic: the reconstruction of logic based on propositions as resources leads to alternatives to traditional syntaxes. The proceedings of the previous three category theory conferences appear as Lecture Notes in Computer Science Volumes 240, 283 and 389.

This book is intended for graduate students and research mathematicians interested in associative rings and algebras.

Introduction to concepts of category theory — categories, functors, natural transformations, the Yoneda lemma, limits and colimits, adjunctions, monads — revisits a broad range of mathematical examples from the categorical perspective. 2016 edition.

Serves as an introduction to higher categories as well as a reference point for many key concepts in the field.