Bicomplex Holomorphic Functions The Algebra Geometry And Analysis Of Bicomplex Numbers Frontiers In Mathematics

The present volume contains a collection of original research articles and expository contributions on recent developments in operator theory and its multifaceted applications. They cover a wide range of themes from the IWOTA 2010 conference held at the TU Berlin, Germany, including spectral theory, function spaces, mathematical system theory, evolution equations and semigroups, and differential and difference operators. The book encompasses new trends and various modern topics in operator theory, and serves as a useful source of information to mathematicians, scientists and engineers.
The articles appeared in this book have been contributed by well-known mathematician and scientists projecting the modern development in the subject. This book covers the following topics Geometry Algebra Functional Analysis Fuzzy Topology Complex Analysis TribologyPostgraduates and researchers working in the areas of mathematics and mathematical sciences will find this book to be of immense value.

The purpose of the volume is to bring forward recent trends of research in hypercomplex analysis. The list of contributors includes first rate mathematicians and young researchers working on several different aspects in quaternionic and Clifford analysis. Besides original research papers, there are papers providing the state-of-the-art of a specific topic, sometimes containing interdisciplinary fields. The intended audience includes researchers, PhD students, postgraduate students who are interested in the field and in possible connection between hypercomplex analysis and other disciplines, including mathematical analysis, mathematical physics, algebra.

* The main treatment is devoted to the analysis of systems of linear partial differential equations (PDEs) with constant coefficients, focusing attention on null solutions of Dirac systems * All the necessary classical material is initially presented * Geared toward graduate students and researchers in (hyper)complex analysis, Clifford analysis, systems of PDEs with constant coefficients, and mathematical physics

This book arose out of original research on the extension of well-established applications of complex numbers related to Euclidean geometry and to the space-time symmetry of two-dimensional Special Relativity. The system of hyperbolic numbers is extensively studied, and a plain exposition of space-time geometry and trigonometry is given. Commutative hypercomplex systems with four unities are studied and attention is drawn to their interesting properties.

The purpose of this book is to develop the foundations of the theory of holomorphicity on the ring of bicomplex numbers. Accordingly, the main focus is on expressing the similarities with, and differences from, the classical theory of one complex variable. The result is an elementary yet comprehensive introduction to the algebra, geometry and analysis of bicomplex numbers. Around the middle of the nineteenth century, several mathematicians (the best known being Sir William Hamilton and Arthur Cayley) became interested in studying number systems that extended the field of complex numbers. Hamilton famously introduced the quaternions, a skew field in real-dimension four, while almost simultaneously James Cockle introduced a commutative four-dimensional real algebra, which was rediscovered in 1892 by Corrado Segre, who referred to his elements as bicomplex numbers. The advantages of commutativity were accompanied by the introduction of zero divisors, something that for a while dampened interest in this subject. In recent years, due largely to the work of G.B. Price, there has been a resurgence of interest in the study of these numbers and, more importantly, in the study of functions defined on the ring of bicomplex numbers, which mimic the behavior of holomorphic functions of a complex

variable. While the algebra of bicomplex numbers is a four-dimensional real algebra, it is useful to think of it as a "complexification" of the field of complex numbers; from this perspective, the bicomplex algebra possesses the properties of a one-dimensional theory inside four real dimensions. Its rich analysis and innovative geometry provide new ideas and potential applications in relativity and quantum mechanics alike. The book will appeal to researchers in the fields of complex, hypercomplex and functional analysis, as well as undergraduate and graduate students with an interest in one- or multidimensional complex analysis.
A rather pretty little book, written in the form of a text but more likely to be read simply for pleasure, in which the author (Professor Emeritus of Mathematics at the U. of Kansas) explores the analog of the theory of functions of a complex variable which comes into being when the complexes are re

The International Society for Analysis, its Applications and Computation (ISAAC) has held its international congresses biennially since 1997. This proceedings volume reports on the progress in analysis, applications and computation in recent years as covered and discussed at the 7th ISAAC Congress. This volume includes papers on partial differential equations, function spaces, operator theory, integral transforms and equations, potential theory, complex analysis and generalizations, stochastic analysis, inverse problems, homogenization, continuum mechanics, mathematical biology and medicine. With over 500 participants from almost 60 countries attending the congress, the book comprises a broad selection of contributions in different topics.

Text on the theory of functions of one complex variable contains, with many elaborations, the subject of the courses and seminars offered by the author over a period of 40 years, and should be considered a source from which a variety of courses can be drawn. In addition to the basic topics in the cl

This volume includes contributions originating from a conference held at Chapman University during November 14-19, 2017. It presents original research by experts in signal processing, linear systems, operator theory, complex and hypercomplex analysis and related topics.

This book presents English translations of Michele Sce's most important works, originally written in Italian during the period 1955-1973, on hypercomplex analysis and algebras of hypercomplex numbers. Despite their importance, these works are not very well known in the mathematics community because of the language they were published in. Possibly the most remarkable instance is the so-called Fueter-Sce mapping theorem, which is a cornerstone of modern hypercomplex analysis, and is not yet understood in its full generality. This volume is dedicated to revealing and describing the framework Sce worked in, at an exciting time when the various generalizations of complex analysis in one variable were still in their infancy. In addition to faithfully translating Sce's papers, the authors discuss their significance and explain their connections to contemporary research in hypercomplex analysis. They also discuss many concrete examples that can serve as a basis for further research. The vast majority of the results presented here will be new to readers, allowing them to finally access the original sources with the benefit of comments from fellow mathematicians active in the field of hypercomplex analysis. As such, the book offers not only an important chapter in the history

of hypercomplex analysis, but also a roadmap for further exciting research in the field.
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analysis.

This book is a collection of short papers from the 11th International ISAAC Congress 2017 in Växjö, Sweden. The papers, written by the best international experts, are devoted to recent results in mathematics with a focus on analysis. The volume provides to both specialists and non-specialists an excellent source of information on the current research in mathematical analysis and its various interdisciplinary applications.

Pseudoanalytic function theory generalizes and preserves many crucial features of complex analytic function theory. The Cauchy-Riemann system is replaced by a much more general first-order system with variable coefficients which turns out to be closely related to important equations of mathematical physics. This relation supplies powerful tools for studying and solving Schrödinger, Dirac, Maxwell, Klein-Gordon and other equations with the aid of complex-analytic methods. The book is dedicated to these recent developments in pseudoanalytic function theory and their applications as well as to multidimensional generalizations. It is directed to undergraduates, graduate students and researchers interested in complex-analytic methods, solution techniques for equations of mathematical physics, partial and ordinary differential equations.

The unifying theme of this book is the interplay among noncommutative geometry, physics, and number theory. The two main objects of investigation are spaces where both the noncommutative and the motivic aspects come to play a role: space-time, where the guiding principle is the problem of developing a quantum theory of gravity, and the space of primes, where one can regard the Riemann Hypothesis as a long-standing problem motivating the development of new geometric tools. The book stresses the relevance of noncommutative geometry in dealing with these two spaces. The first part of the book deals with quantum field theory and the geometric structure of renormalization as a Riemann-Hilbert correspondence. It also presents a model of elementary particle physics based on noncommutative geometry. The main result is a complete derivation of the full Standard Model Lagrangian from a very simple mathematical input. Other topics covered in the first part of the book are a noncommutative geometry model of dimensional regularization and its role in anomaly computations, and a brief introduction to motives and their conjectural relation to quantum field theory. The second part of the book gives an interpretation of the Weil explicit formula as a trace formula and a spectral realization of the zeros of the Riemann zeta function. This is based on the noncommutative geometry of the adèle class space, which is also described as the space of commensurability classes of Q-lattices, and is dual to a noncommutative motive (endomotive) whose cyclic homology provides a general setting for spectral realizations of zeros of L-functions. The quantum statistical mechanics of the space of Q-lattices, in one and two dimensions, exhibits spontaneous symmetry breaking. In the low-temperature regime, the equilibrium states of the corresponding systems are related to points of classical moduli spaces and the symmetries to the class field theory of the field of rational numbers and of imaginary quadratic fields, as well as to the automorphisms of the field of modular functions. The book ends with a set of analogies between the noncommutative geometries underlying the mathematical formulation of the Standard Model minimally coupled to gravity and the moduli spaces of Q-lattices used in the study of the zeta function.

This volume is intended to collect important research results to the lectures and discussions which took Place in Rome, at the INdAM Workshop on Different Notions of Regularity for Functions of Quaternionic Variables in September 2010. This volume will collect recent and new results, which are connected to the topic covered during the workshop. The work aims at bringing together international leading specialists in the field of Quaternionic and Clifford Analysis, as well as young researchers interested in the subject, with the idea of presenting and discussing recent results, analyzing new trends and techniques in the area and, in general, of promoting scientific collaboration. Particular attention is paid to the presentation of different notions of regularity for functions of hypercomplex variables, and to the study of the main features of the theories that they originate.

"Provides a thorough introduction to the algebraic theory of systems of differential equations, as developed by the Japanese school of M. Sato and his colleagues. Features a complete review of hyperfunction-microfunction theory and the theory of D-modules. Strikes the perfect balance between analytic and algebraic aspects."

This book provides a new mathematical theory for the treatment of an ample series of spatial problems of electrodynamics, particle physics, quantum mechanics and elasticity theory. This technique proves to be as powerful for solving the spatial problems of mathematical physics as complex analysis is for solving planar problems. The main analytic tool of the book, a non-harmonic version of hypercomplex analysis recently developed by the authors, is presented in detail. There are given applications of this theory to the boundary value problems of electrodynamics and elasticity theory as well as to the problem of quark confinement. A new approach to the linearization of special classes of the self-duality equation is also considered. Detailed proofs are given throughout. The book contains an extensive bibliography on closely related topics. This book will be of particular interest to academic and professional specialists and students in mathematics and physics who are interested in integral representations for partial differential equations. The book is self-contained and could be used as a main reference for special course seminars on the subject.

Contains selected papers from the ISAAC conference 2007 and invited contributions. This book covers various topics that represent the main streams of research in hypercomplex analysis as well as the expository articles. It is suitable for researchers and postgraduate students in various areas of mathematical analysis.

This book provides the foundations for a rigorous theory of functional analysis with bicomplex scalars. It begins with a detailed study of bicomplex and hyperbolic numbers and then defines the notion of bicomplex modules. After introducing a number of norms and inner products on such modules (some of which appear in this volume for the first time), the authors develop the theory of linear functionals and linear operators on bicomplex modules. All of this may serve for many different developments, just like the usual functional analysis with complex scalars and in this book it serves as the foundational material for the construction and study of a bicomplex version of the well known Schur analysis.

Hypercomplex analysis is the extension of complex analysis to higher dimensions where the concept of a holomorphic function is substituted by the concept of a monogenic function. In recent decades this theory has come to the forefront of higher dimensional analysis. There are several approaches to this: quaternionic analysis which merely uses quaternions, Clifford analysis which relies on Clifford algebras, and generalizations of complex variables to higher dimensions such as split-complex variables. This book includes a selection of papers presented at the session on quaternionic and hypercomplex analysis at the ISAAC conference 2013 in Krakow, Poland. The topics covered represent new perspectives and current trends in hypercomplex analysis and applications to mathematical physics, image analysis and processing, and mechanics.

This book concisely presents a broad range of models and theories on social systems. Because of the huge spectrum of topics involving social systems, various issues related to Mathematics, Statistics, Teaching, Social Science, and Economics are discussed. In an effort to introduce the subject to a wider audience, this volume, part of the series “Studies in

Systems, Decision and Control", equally addresses the needs of mathematicians, statisticians, sociologists and philosophers. The studies examined here are divided into four parts. The first part, "Perusing the Minds Behind Scientific Discoveries", traces the winding path of Syamal K. Sen and Ravi P. Agarwal's scholarship throughout history, and most importantly, the thought processes that allowed each of them to master their subject. The second part covers "Theories in Social Systems" and the third discusses "Models in Social Systems", while the fourth and final part is dedicated to "Mathematical Methods in the Social Sciences". Given its breadth of coverage, the book will offer inquisitive readers a valuable point of departure for exploring these rich, vast, and ever-expanding fields of knowledge.

This book presents applications of hypercomplex analysis to boundary value and initial-boundary value problems from various areas of mathematical physics. Given that quaternion and Clifford analysis offer natural and intelligent ways to enter into higher dimensions, it starts with quaternion and Clifford versions of complex function theory including series expansions with Appell polynomials, as well as Taylor and Laurent series. Several necessary function spaces are introduced, and an operator calculus based on modifications of the Dirac, Cauchy-Fueter, and Teodorescu operators and different decompositions of quaternion Hilbert spaces are proved. Finally, hypercomplex Fourier transforms are studied in detail. All this is then applied to first-order partial differential equations such as the Maxwell equations, the Carleman-Bers-Vekua system, the Schrödinger equation, and the Beltrami equation. The higher-order equations start with Riccati-type equations. Further topics include spatial fluid flow problems, image and multi-channel processing, image diffusion, linear scale invariant filtering, and others. One of the highlights is the derivation of the three-dimensional Kolosov-Mushkelishvili formulas in linear elasticity. Throughout the book the authors endeavor to present historical references and important personalities. The book is intended for a wide audience in the mathematical and engineering sciences and is accessible to readers with a basic grasp of real, complex, and functional analysis.

This book gathers contributions written by Daniel Alpay’s friends and collaborators. Several of the papers were presented at the International Conference on Complex Analysis and Operator Theory held in honor of Professor Alpay’s 60th birthday at Chapman University in November 2016. The main topics covered are complex analysis, operator theory and other areas of mathematics close to Alpay’s primary research interests. The book is recommended for mathematicians from the graduate level on, working in various areas of mathematical analysis, operator theory, infinite dimensional analysis, linear systems, and stochastic processes.

The fundamental theorem of algebra states that any complex polynomial must have a complex root. This book examines three pairs of proofs of the theorem from three different areas of mathematics: abstract algebra, complex analysis and topology. The first proof in each pair is fairly straightforward and depends only on what could be considered elementary mathematics. However, each of these first proofs leads to more general results from which the fundamental theorem can be deduced as a direct consequence. These general results constitute the second proof in each pair. To arrive at each of the proofs, enough of the general theory of each relevant area is developed to understand the proof. In addition to the proofs and techniques themselves, many applications such as the insolvability of the quintic and the transcendence of e and pi are presented. Finally, a series of appendices give six additional proofs including a version of Gauss'original first proof. The book is intended for junior/senior level undergraduate mathematics students or first year graduate students, and would make an ideal "capstone" course in mathematics.

' In the last decade, the development of new ideas in quantum theory, including geometric and deformation quantization, the non-Abelian Berry's geometric factor, super- and BRST symmetries, non-commutativity, has called into play the geometric techniques based on the deep interplay between algebra, differential geometry and topology. The book aims at being a guide to advanced differential geometric and topological methods in quantum mechanics. Their main peculiarity lies in the fact that geometry in quantum theory speaks mainly the algebraic language of rings, modules, sheaves and categories. Geometry is by no means the primary scope of the book, but it underlies many ideas in modern quantum physics and provides the most advanced schemes of quantization. Contents:Commutative GeometryClassical Hamiltonian SystemsAlgebraic QuantizationGeometry of Algebraic QuantizationGeometric QuantizationSupergeometryDeformation QuantizationNon-Commutative GeometryGeometry of Quantum Groups Readership: Theoreticians and mathematicians of postgraduate and research level. Keywords:Algebraic Quantum Theory;Poisson Manifold;Hilbert Manifold;Geometric Quantization;Deformation Quantization;Supergeometry;Noncommutative Geometry;Constraint System;Quantum GroupKey Features:The book collects all the advanced methods of quantization in the last decadeIt presents in a compact way all the necessary up to date mathematical tools to be used in studying quantum problems.Reviews:“This book is well-written and I am convinced that it will be useful to all those interested in quantum theory.”Zentralblatt MATH “With respect to a propsective reader having a reasonably good background in mathematics, the notions, concepts, etc, are introduced in a self-contained but condensed manner … The book gives a very helpful supply of mathematical tools needed by a theoretical or mathematical physicist to effect entry into some of the new directions in theoretical physics. Also, a mathematician might appreciate the condensed presentation of definitions and results in one of the modern fields of mathematics for which one may be seeking an overview."Mathematical Reviews '

First Joint International Meeting of the Israel Mathematical Union and the Mexican Mathematical Society Program

From the Preface (1960): ``This book is devoted to an account of one of the branches of functional analysis, the theory of commutative normed rings, and the principal applications of that theory. It is based on [the authors'] paper written ... in 1940, hard on the heels of the initial period of the development of this theory ... ``The book consists of three parts. Part one, concerned with the theory of commutative normed rings and divided into two chapters; the first containing foundations of the theory and the second dealing with more special problems. Part two deals with applications to harmonic analysis and is divided into three chapters. The first chapter discusses the ring of absolutely integrable functions on a line with convolution as multiplication and finds the maximal ideals of this ring and some of its analogues. In the next chapter, these results are carried over to arbitrary commutative locally compact groups and they are made the foundation of the construction of harmonic analysis and the theory of characters. A new feature here is the construction of an invariant measure on the group of characters and a proof of the inversion formula for Fourier transforms that is not based on theorems on the representation of positive-definite functions or positive functionals ... The last chapter of the second part--the most specialized of all the chapters--is devoted to the investigation of the ring of functions of bounded variation on a line with multiplication defined as convolution, including the complete description of the maximal ideals of this ring. The third part of the book is devoted to the discussion of two important classes of rings of functions: regular rings and rings with uniform convergence. The first of the chapters essentially studies the structure of ideals in regular rings. The chapter ends with an example of a ring of functions having closed ideals that cannot be represented as the intersections of maximal ideals. The second chapter discusses the ring $C(S)$ of all bounded continuous complex functions on completely regular spaces S and various of its subrings ... ``Since noncommutative normed rings with an involution are important for group-theoretical applications, the paper by I. M. Gelfand and N. A. Naimark, `Normed Rings with an Involution and their Representations', is reproduced at the end of the book, slightly abridged, in the form of an appendix ... This monograph also contains an account of the foundations of the theory of commutative normed rings without, however, touching upon the majority of its analytic applications ... ``The reader [should] have knowledge of the elements of the theory of normed spaces and of set-theoretical topology. For an understanding of the fourth chapter, [the reader should] also know what a topological group is. It stands to reason that the basic concepts of the theory of measure and of the Lebesgue integral are also assumed to be known ... ''

Noncommutative Geometry is one of the most deep and vital research subjects of present-day Mathematics. Its development, mainly due to Alain Connes, is providing an increasing number of applications and deeper insights for instance in Foliations, K-Theory, Index Theory, Number Theory but also in Quantum

Physics of elementary particles. The purpose of the Summer School in Martina Franca was to offer a fresh invitation to the subject and closely related topics; the contributions in this volume include the four main lectures, cover advanced developments and are delivered by prominent specialists.

This book is the result of many years of research in Non-Euclidean Geometries and Geometry of Lie groups, as well as teaching at Moscow State University (1947- 1949), Azerbaijan State University (Baku) (1950-1955), Kolomna Pedagogical Col lege (1955-1970), Moscow Pedagogical University (1971-1990), and Pennsylvania State University (1990-1995). My first books on Non-Euclidean Geometries and Geometry of Lie groups were written in Russian and published in Moscow: Non-Euclidean Geometries (1955) [Ro1] , Multidimensional Spaces (1966) [Ro2] , and Non-Euclidean Spaces (1969) [Ro3]. In [Ro1] I considered non-Euclidean geometries in the broad sense, as geometry of simple Lie groups, since classical non-Euclidean geometries, hyperbolic and elliptic, are geometries of simple Lie groups of classes Bn and D , and geometries of complex n and quaternionic Hermitian elliptic and hyperbolic spaces are geometries of simple Lie groups of classes An and en. [Ro1] contains an exposition of the geometry of classical real non-Euclidean spaces and their interpretations as hyperspheres with identified antipodal points in Euclidean or pseudo-Euclidean spaces, and in projective and conformal spaces. Numerous interpretations of various spaces different from our usual space allow us, like stereoscopic vision, to see many traits of these spaces absent in the usual space.

This book, intended to commemorate the work of Paul Dirac, highlights new developments in the main directions of Clifford analysis. Just as complex analysis is based on the algebra of the complex numbers, Clifford analysis is based on the geometric Clifford algebras. Many methods and theorems from complex analysis generalize to higher dimensions in various ways. However, many new features emerge in the process, and much of this work is still in its infancy. Some of the leading mathematicians working in this field have contributed to this book in conjunction with “Clifford Analysis and Related Topics: a conference in honor of Paul A.M. Dirac,” which was held at Florida State University, Tallahassee, on December 15-17, 2014. The content reflects talks given at the conference, as well as contributions from mathematicians who were invited but were unable to attend. Hence much of the mathematics presented here is not only highly topical, but also cannot be found elsewhere in print. Given its scope, the book will be of interest to mathematicians and physicists working in these areas, as well as students seeking to catch up on the latest developments.

"Basic Noncommutative Geometry provides an introduction to noncommutative geometry and some of its applications. The book can be used either as a textbook for a graduate course on the subject or for self-study. It will be useful for graduate students and researchers in mathematics and theoretical physics and all those who are interested in gaining an understanding of the subject. One feature of this book is the wealth of examples and exercises that help the reader to navigate through the subject. While background material is provided in the text and in several appendices, some familiarity with basic notions of functional analysis, algebraic topology, differential geometry and homological algebra at a first year graduate level is helpful. Developed by Alain Connes since the late 1970s, noncommutative geometry has found many applications to long-standing conjectures in topology and geometry and has recently made headways in theoretical physics and number theory. The book starts with a detailed description of some of the most pertinent algebra-geometry correspondences by casting geometric notions in algebraic terms, then proceeds in the second chapter to the idea of a noncommutative space and how it is constructed. The last two chapters deal with homological tools: cyclic cohomology and Connes-Chern characters in K-theory and K-homology, culminating in one commutative diagram expressing the equality of topological and analytic index in a noncommutative setting. Applications to integrality of noncommutative topological invariants are given as well."--Publisher's description.

Bicomplex Holomorphic FunctionsThe Algebra, Geometry and Analysis of Bicomplex NumbersBirkhäuser

This volume contains the contributions by the participants in the eight of a series workshops in complex analysis, differential geometry and mathematical physics and related areas.Active specialists in mathematical physics contribute to the volume, providing not only significant information for researchers in the area but also interesting mathematics for non-specialists and a broader audience. The contributions treat topics including differential geometry, partial differential equations, integrable systems and mathematical physics.