# Bernoulli Numbers And Zeta Functions Springer Monographs In Mathematics

This book explores the rich and elegant interplay between the two main currents of mathematics, the continuous and the discrete. Such fundamental notions in discrete mathematics as induction, recursion, combinatorics, number theory, discrete probability, and the algorithmic point of view as a unifying principle are continually explored as they interact with traditional calculus.

Zeta and q-Zeta Functions and Associated Series and Integrals is a thoroughly revised, enlarged and updated version of Series Associated with the Zeta and Related Functions. Many of the chapters and sections of the book have been significantly modified or rewritten, and a new chapter on the theory and applications of the basic (or q-) extensions of various special functions is included. This book will be invaluable because it covers not only detailed and systematic presentations of the theory and applications of the various methods and techniques used in dealing with many different classes of series and integrals associated with the Zeta and related functions, but stimulating historical accounts of a large number of problems and well-classified tables of series and integrals. Detailed and systematic presentations of the theory and applications of the various methods and techniques used in dealing with many different classes of series and integrals associated with the Zeta and related functions

This is the first introductory book on multiple zeta functions and multiple polylogarithms which are the generalizations of the Riemann zeta function and the classical polylogarithms, respectively, to the multiple variable setting. It contains all the basic concepts and the important properties of these functions and their special values. This book is aimed at graduate students, mathematicians and physicists who are interested in this current active area of research. The book will provide a detailed and comprehensive introduction to these objects, their fascinating properties and interesting relations to other mathematical subjects, and various generalizations such as their q-analogs and their finite versions (by taking partial sums modulo suitable prime powers). Historical notes and exercises are provided at the end of each chapter. Contents:Multiple Zeta FunctionsMultiple Polylogarithms (MPLs)Multiple Zeta Values (MZVs)Drinfeld Associator and Single-Valued MZVsMultiple Zeta Value IdentitiesSymmetrized Multiple Zeta Values (SMZVs)Multiple Harmonic Sums (MHSs) and Alternating VersionFinite Multiple Zeta Values and Finite Euler Sumsq-Analogs of Multiple Harmonic (Star) Sums Readership: Advanced undergraduates and graduate students in mathematics, mathematicians interested in multiple zeta values. Key Features:For the first time, a detailed explanation of the theory of multiple zeta values is given in book form along with numerous illustrations in explicit examplesThe book provides for the first time a comprehensive introduction to multiple polylogarithms and their special values at roots of unity, from the basic definitions to the more advanced topics in current active researchThe book contains a few quite intriguing results relating the special values of multiple zeta functions and multiple polylogarithms to other branches of mathematics and physics, such as knot theory and the theory of motivesMany exercises contain supplementary materials which deepens the reader's understanding of the main text

Since the publication of the first edition of this work, considerable progress has been made in many of the questions examined. This edition has been updated and enlarged, and the bibliography has been revised. The variety of topics covered here includes divisibility, diophantine equations, prime numbers (especially Mersenne and Fermat primes), the basic arithmetic functions, congruences, the quadratic reciprocity law, expansion of real numbers into decimal fractions, decomposition of integers into sums of powers, some other problems of the additive theory of numbers and the theory of Gaussian integers.

This text covers exponential integrals and sums, 4th power moment, zero-free region, mean value estimates over short intervals, higher power moments, omega results, zeros on the critical line, zero-density estimates, and more. 1985 edition.

Designed as a reference work and also as a graduate-level textbook, this volume presents an up-to-date and comprehensive account of the theories and applications of the various methods and techniques used in dealing with problems involving closed-form evaluations of (and representations of the Riemann Zeta function at positive integer arguments as) numerous families of series associated with the Riemann Zeta function, the Hurwitz Zeta function, and their extensions and generalizations such as Lerch's transcendent (or the Hurwitz-Lerch Zeta function). Audience: This book is intended for professional mathematicians and graduate students in mathematical sciences (both pure and applied).

This book collects more than thirty contributions in memory of Wolfgang Schwarz, most of which were presented at the seventh International Conference on Elementary and Analytic Number Theory (ELAZ), held July 2014 in Hildesheim, Germany. Ranging from the theory of arithmetical functions to diophantine problems, to analytic aspects of zeta-functions, the various research and survey articles cover the broad interests of the well-known number theorist and cherished colleague Wolfgang Schwarz (1934-2013), who contributed over one hundred articles on number theory, its history and related fields. Readers interested in elementary or analytic number theory and related fields will certainly find many fascinating topical results among the contributions from both respected mathematicians and up-and-coming young researchers. In addition, some biographical articles highlight the life and mathematical works of Wolfgang Schwarz.

The theory of modular forms is a fundamental tool used in many areas of mathematics and physics. It is also a very concrete and “fun” subject in itself and abounds with an amazing number of surprising identities. This comprehensive textbook, which includes numerous exercises, aims to give a complete picture of the classical aspects of the subject, with an emphasis on explicit formulas. After a number of motivating examples such as elliptic functions and theta functions, the modular group, its subgroups, and general aspects of holomorphic and nonholomorphic modular forms are explained, with an emphasis on explicit examples. The heart of the book is the classical theory developed by Hecke and continued up to the Atkin–Lehner–Li theory of newforms and including the theory of Eisenstein series, Rankin–Selberg theory, and a more general theory of theta series including the Weil representation. The final chapter explores in some detail more general types of modular forms such as half-integral weight, Hilbert, Jacobi, Maass, and Siegel modular forms. Some “gems” of the book are an immediately implementable trace formula for Hecke operators, generalizations of Haberland's formulas for the computation of Petersson inner products, W. Li's little-known theorem on the diagonalization of the full space of modular forms, and explicit algorithms due to the second author for computing Maass forms. This book is essentially self-contained, the necessary tools such as gamma and Bessel functions, Bernoulli numbers, and so on being given in a separate chapter.

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Algorithmics of Nonuniformity is a solid presentation about the analysis of algorithms, and the data structures that support them. Traditionally, algorithmics have been approached either via a probabilistic view or an analytic approach. The authors adopt both approaches and bring them together to get the best of both worlds and benefit from the advantage of each approach. The text examines algorithms that are designed to handle general data—sort any array, find the median of any numerical set, and identify patterns in any setting. At the same time, it evaluates "average" performance, "typical" behavior, or in mathematical terms, the expectations of the random variables that describe their operations. Many exercises are presented, which are essential since they convey additional material complementing the content of the chapters. For this reason, the solutions are more than mere answers, but explain and expand upon related concepts, and motivate further work by the reader. Highlights: A unique book that merges probability with analysis of algorithms Approaches analysis of algorithms from the angle of uniformity Non-uniformity makes more realistic models of real-life scenarios possible Results can be applied to many applications Includes many exercises of various levels of difficulty About the Authors: Micha Hofri is a Professor of Computer Science, and former department head at Worcester Polytechnic Institute. He holds a Ph.D. of Industrial Engineering (1972), all from Technion, the Israel Institute of Technology. He has 39 publications in Mathematics. Hosam Mahmoud is a Professor at, the Department of Statistics at George Washington University in Washington D.C., where he used to be the former chair. He holds an Ph.D. in Computer Science from Ohio State University. He is on the editorial board of five academic journals.

Recipient of the Mathematical Association of America's Beckenbach Book Prize in 2008! Leonhard Euler was one of the most prolific mathematicians that have ever lived. This book examines the huge scope of mathematical areas explored and developed by Euler, which includes number theory, combinatorics, geometry, complex variables and many more. The information known to Euler over 300 years ago is discussed, and many of his advances are reconstructed. Readers will be left in no doubt about the brilliance and pervasive influence of Euler's work.

An undergraduate-level 2003 introduction whose only prerequisite is a standard calculus course.

Generatingfunctionology provides information pertinent to generating functions and some of their uses in discrete mathematics. This book presents the power of the method by giving a number of examples of problems that can be profitably thought about from the point of view of generating functions. Organized into five chapters, this book begins with an overview of the basic concepts of a generating function. This text then discusses the different kinds of series that are widely used as generating functions. Other chapters explain how to make much more precise estimates of the sizes of the coefficients of power series based on the analyticity of the function that is represented by the series. This book discusses as well the applications of the theory of generating functions to counting problems. The final chapter deals with the formal aspects of the theory of generating functions. This book is a valuable resource for mathematicians and students.

This survey explores the history of the arithmetical triangle, from its roots in Pythagorean arithmetic, Hindu combinatorics, and Arabic algebra to its influence on Newton and Leibniz as well as modern-day mathematicians.

This book presents a method for evaluating Selberg zeta functions via transfer operators for the full modular group and its congruence subgroups with characters. Studying zeros of Selberg zeta functions for character deformations allows us to access the discrete spectra and resonances of hyperbolic Laplacians under both singular and non-singular perturbations. Areas in which the theory has not yet been sufficiently developed, such as the spectral theory of transfer operators or the singular perturbation theory of hyperbolic Laplacians, will profit from the numerical experiments discussed in this book. Detailed descriptions of numerical approaches to the spectra and eigenfunctions of transfer operators and to computations of Selberg zeta functions will be of value to researchers active in analysis, while those researchers focusing more on numerical aspects will benefit from discussions of the analytic theory, in particular those concerning the transfer operator method and the spectral theory of hyperbolic spaces.

The first edition of this work has become the standard introduction to the theory of p-adic numbers at both the advanced undergraduate and beginning graduate level. This second edition includes a deeper treatment of p-adic functions in Ch. 4 to include the Iwasawa logarithm and the p-adic gamma-function, the rearrangement and addition of some exercises, the inclusion of an extensive appendix of answers and hints to the exercises, as well as numerous clarifications.

This book, in honor of Hari M. Srivastava, discusses essential developments in mathematical research in a variety of problems. It contains thirty-five articles, written by eminent scientists from the international mathematical community, including both research and survey works. Subjects covered include analytic number theory, combinatorics, special sequences of numbers and polynomials, analytic inequalities and applications, approximation of functions and quadratures, orthogonality and special and complex functions. The mathematical results and open problems discussed in this book are presented in a simple and self-contained manner. The book contains an overview of old and new results, methods, and theories toward the solution of longstanding problems in a wide scientific field, as well as new results in rapidly progressing areas of research. The book will be useful for researchers and graduate students in the fields of mathematics, physics and other computational and applied sciences.

Graph theory meets number theory in this stimulating book. Ihara zeta functions of finite graphs are reciprocals of polynomials, sometimes in several variables. Analogies abound with number-theoretic functions such as Riemann/Dedekind zeta functions. For example, there is a Riemann hypothesis (which may be false) and prime number theorem for graphs. Explicit constructions of graph coverings use Galois theory to generalize Cayley and Schreier graphs. Then non-isomorphic simple graphs with the same zeta are produced, showing you cannot hear the shape of a graph. The spectra of matrices such as the adjacency and edge adjacency matrices of a graph are essential to the plot of this book, which makes connections with quantum chaos and random matrix theory, plus expander/Ramanujan graphs of interest in computer science. Created for beginning graduate students, the book will also appeal to researchers. Many well-chosen illustrations and exercises, both theoretical and computer-based, are included throughout.

Multiple Dirichlet Series, L-functions and Automorphic Forms gives the latest advances in the rapidly developing subject of Multiple Dirichlet Series, an area with origins in the theory of automorphic forms that exhibits surprising and deep connections to crystal graphs and mathematical physics. As such, it represents a new way in which areas including number theory, combinatorics, statistical mechanics, and quantum groups are seen to fit together. The volume also includes papers on automorphic forms and L-functions and related number-theoretic topics. This volume will be a valuable resource for graduate students and researchers in number theory, combinatorics, representation theory, mathematical physics, and special functions. Contributors: J. Beineke, B. Brubaker, D. Bump, G. Chinta, G. Cornelissen, C.A. Diaconu, S. Frechette, S. Friedberg, P. Garrett, D. Goldfeld, P.E. Gunnells, B. Heim, J. Hundley, D. Ivanov, Y. Komori, A.V. Kontorovich, O. Lorscheid, K. Matsumoto, P.J. McNamara, S.J. Patterson, M. Suzuki, H. Tsumura.

This is a first-ever textbook written in English about the theory of modular forms and Jacobi forms of several variables. It contains the classical theory as well as a new theory on Jacobi forms over Cayley numbers developed by the author from 1990 to 2000. Applications to the classical Euler sums are of special interest to those who are eager to evaluate double Euler sums or more general multiple zeta values. The celebrated sum formula proved by Granville in 1997 is generalized to a more general form here.

The mathematical combinatorics is a subject that applying combinatorial notion to all mathematics and all sciences for understanding the reality of things in the universe. The International J. Mathematical Combinatorics is a fully refereed international journal, sponsored by the MADIS of Chinese Academy of Sciences and published in USA quarterly, which publishes original research papers and survey articles in all aspects of mathematical combinatorics, Smarandache multi-spaces, Smarandache geometries, non-Euclidean geometry, topology and their applications to other sciences.

The book includes several survey articles on prime numbers, divisor problems, and Diophantine equations, as well as research papers on various aspects of analytic number theory such as additive problems, Diophantine approximations and the theory of zeta and L-function. Audience: Researchers and graduate students interested in recent development of number theory.

This text originated as a lecture delivered November 20, 1984, at Queen's University, in the undergraduate colloquium senes. In another colloquium lecture, my colleague Morris Orzech, who had consulted the latest edition of the Guinness Book of Records, reminded me very gently that the most "innumerate" people of the world are of a certain trible in Mato Grosso, Brazil. They do not even have a word to express the number "two" or the concept of plurality. "Yes, Morris, I'm from Brazil, but my book will contain numbers different from ·one."' He added that the most boring 800-page book is by two Japanese mathematicians (whom I'll not name) and consists of about 16 million decimal digits of the number Te. "I assure you, Morris, that in spite of the beauty of the appar ent randomness of the decimal digits of Te, I'll be sure that my text will include also some words." And then I proceeded putting together the magic combina tion of words and numbers, which became The Book of Prime Number Records. If you have seen it, only extreme curiosity could impel you to have this one in your hands. The New Book of Prime Number Records differs little from its predecessor in the general planning. But it contains new sections and updated records.

The subject of special functions is rich and expanding continuously with the emergence of new problems encountered in engineering and applied science applications. The development of computational techniques and the rapid growth in computing power have increased the importance of the special functions and their formulae for analytic representations. However, problems remain, particularly in heat conduction, astrophysics, and probability theory, whose solutions seem to defy even the most general classes of special functions. On a Class of Incomplete Gamma Functions with Applications introduces a class of special functions, developed by the authors, useful in the analytic study of several heat conduction problems. It presents some basic properties of these functions, including their recurrence relations, special cases, asymptotic representations, and integral transform relationships. The authors explore applications of these generalized functions to problems in transient heat conduction, special cases of laser sources, and problems associated with heat transfer in human tissues. They also discuss applications to astrophysics, probability theory, and other problems in theory of functions and present a fundamental solution to time-dependent laser sources with convective-type boundary conditions. Appendices include an introduction to heat conduction, Fourier conduction, a table of Laplace transforms, and well-known results regarding the improper integrals. Filled with tabular and graphical representations for applications, this monograph offers a unique opportunity to add to your mathematical toolbox a new and useful class of special functions.

Two major subjects are treated in this book. The main one is the theory of Bernoulli numbers and the other is the theory of zeta functions. Historically, Bernoulli numbers were introduced to give formulas for the sums of powers of consecutive integers. The real reason that they are indispensable for number theory, however, lies in the fact that special values of the Riemann zeta function can be written by using Bernoulli numbers. This leads to more advanced topics, a number of which are treated in this book: Historical remarks on Bernoulli numbers and the formula for the sum of powers of consecutive integers; a formula for Bernoulli numbers by Stirling numbers; the Clausen–von Staudt theorem on the denominators of Bernoulli numbers; Kummer's congruence between Bernoulli numbers and a related theory of p-adic measures; the Euler–Maclaurin summation formula; the functional equation of the Riemann zeta function and the Dirichlet L functions, and their special values at suitable integers; various formulas of exponential sums expressed by generalized Bernoulli numbers; the relation between ideal classes of orders of quadratic fields and equivalence classes of binary quadratic forms; class number formula for positive definite binary quadratic forms; congruences between some class numbers and Bernoulli numbers; simple zeta functions of prehomogeneous vector spaces; Hurwitz numbers; Barnes multiple zeta functions and their special values; the functional equation of the doub le zeta functions; and poly-Bernoulli numbers. An appendix by Don Zagier on curious and exotic identities for Bernoulli numbers is also supplied. This book will be enjoyable both for amateurs and for professional researchers. Because the logical relations between the chapters are loosely connected, readers can start with any chapter depending on their interests. The expositions of the topics are not always typical, and some parts are completely new.

The author demonstrates that the Dirichlet series representation of the Riemann zeta function converges geometrically at the roots in the critical strip. The Dirichlet series parts of the Riemann zeta function diverge everywhere in the critical strip. It has therefore been assumed for at least 150 years that the Dirichlet series representation of the zeta function is useless for characterization of the non-trivial roots. The author shows that this assumption is completely wrong. Reduced, or simplified, asymptotic expansions for the terms of the zeta function series parts are equated algebraically with reduced asymptotic expansions for the terms of the zeta function series parts with reflected argument, constraining the real parts of the roots of both functions to the critical line. Hence, the Riemann hypothesis is correct. Formulae are derived and solved numerically, yielding highly accurate values of the imaginary parts of the roots of the zeta function.

In the spirit of Alladi Ramakrishnan’s profound interest and contributions to three fields of science — Mathematics, Statistics, and Physics — this volume contains invited surveys and research articles from prominent members of these communities who also knew Ramakrishnan personally and greatly respected his influence in these areas of science. Historical photos, telegrams, and biographical narratives of Alladi Ramakrishnan’s illustrious career of special interest are included as well.

Upon publication, the first edition of the CRC Concise Encyclopedia of Mathematics received overwhelming accolades for its unparalleled scope, readability, and utility. It soon took its place among the top selling books in the history of Chapman & Hall/CRC, and its popularity continues unabated. Yet also unabated has been the d

This volume contains a valuable collection of articles presented at a conference on Automorphic Forms and Zeta Functions in memory of Tsuneo Arakawa, an eminent researcher in modular forms in several variables and zeta functions. The book begins with a review of his works, followed by 16 articles by experts in the fields including H Aoki, R Berndt, K Hashimoto, S Hayashida, Y Hironaka, H Katsurada, W Kohnen, A Krieg, A Murase, H Narita, T Oda, B Roberts, R Schmidt, R Schulze-Pillot, N Skoruppa, T Sugano, and D Zagier. A variety of topics in the theory of modular forms and zeta functions are covered: Theta series and the basis problems, Jacobi forms, automorphic forms on Sp(1, q), double zeta functions, special values of zeta and L-functions, many of which are closely related to Arakawa's works. This collection of papers illustrates Arakawa's contributions and the current trends in modular forms in several variables and related zeta functions. Contents: Tsuneo Arakawa and His Works; Estimate of the Dimensions of Hilbert Modular Forms by Means of Differential Operator (H Aoki); Marsden-Weinstein Reduction, Orbits and Representations of the Jacobi Group (R Berndt); On Eisenstein Series of Degree Two for Squarefree Levels and the Genus Version of the Basis Problem I (S Bocherer); Double Zeta Values and Modular Forms (H Gangl et al.); Type Numbers and Linear Relations of Theta Series for Some General Orders of Quaternion Algebras (K Hashimoto); Skewholomorphic Jacobi Forms of Higher Degree (S Hayashida); A Hermitian Analog of the Schottky Form (M Hentschel & A Krieg); The Siegel Series and Spherical Functions on O(2n)/(O(n) x O(n)) (Y Hironaka & F Sati); Koecher-Maa Series for Real Analytic Siegel Eisenstein Series (T Ibukiyama & H Katsurada); A Short History on Investigation of the Special Values of Zeta and L-Functions of Totally Real Number Fields (T Ishii & T Oda); Genus Theta Series, Hecke Operators and the Basis Problem for Eisenstein Series (H Katsurada & R Schulze-Pillot); The Quadratic Mean of Automorphic L-Functions (W Kohnen et al.); Inner Product Formula for Kudla Lift (A Murase & T Sugano); On Certain Automorphic Forms of Sp(1,q) (Arakawa's Results and Recent Progress) (H Narita); On Modular Forms for the Paramodular Group (B Roberts & R Schmidt); SL(2,Z)-Invariant Spaces Spanned by Modular Units (N-P Skoruppa & W Eholzer). Readership: Researchers and graduate students in number theory or representation theory as well as in mathematical physics or combinatorics.

This volume contains the proceedings of the workshop held at the DIMACS Center of Rutgers University (Piscataway, NJ) on Unusual Applications of Number Theory. Standard applications of number theory are to computer science and cryptology. In this volume, well-known number theorist, Melvyn B. Nathanson, gathers articles from the workshop on other, less standard applications in number theory, as well as topics in number theory with potential applications in science and engineering. The material is suitable for graduate students and researchers interested in number theory and its applications.

This text on a central area of number theory covers p-adic L-functions, class numbers, cyclotomic units, Fermat’s Last Theorem, and Iwasawa’s theory of Z_p-extensions. This edition contains a new chapter on the work of Thaine, Kolyvagin, and Rubin, including a proof of the Main Conjecture, as well as a chapter on other recent developments, such as primality testing via Jacobi sums and Sinnott’s proof of the vanishing of Iwasawa’s f-invariant.

This volume provides a systematic survey of almost all the equivalent assertions to the functional equations - zeta symmetry - which zeta-functions satisfy, thus streamlining previously published results on zeta-functions. The equivalent relations are given in the form of modular relations in Fox H-function series, which at present include all that have been considered as candidates for ingredients of a series. The results are presented in a clear and simple manner for readers to readily apply without much knowledge of zeta-functions. This volume aims to keep a record of the 150-year-old heritage starting from Riemann on zeta-functions, which are ubiquitous in all mathematical sciences, wherever there is a notion of the norm. It provides almost all possible equivalent relations to the zeta-functions without requiring a reader's deep knowledge on their definitions. This can be an ideal reference book for those studying zeta-functions.

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A new edition of a classical treatment of elliptic and modular functions with some of their number-theoretic applications, this text offers an updated bibliography and an alternative treatment of the transformation formula for the Dedekind eta function. It covers many topics, such as Hecke's theory of entire forms with multiplicative Fourier coefficients, and the last chapter recounts Bohr's theory of equivalence of general Dirichlet series.