Automorphic Forms And L Functions For The Group Gl Nr

Automorphic Forms and L-Functions for the Group GL(n,R)Cambridge University Press
This book presents a treatment of the theory of L-functions developed via the theory of Eisenstein series and their Fourier coefficients. The author is a co-developer of the important Langlands-Shahidi method. This account of the theory is ideal for graduate students and researchers interested in the Langlands program in automorphic forms and its connections with number theory.
This book presents a collection of carefully refereed research articles and lecture notes stemming from the Conference "Automorphic Forms and L-Functions", held at the University of Heidelberg in 2016. The theory of automorphic forms and their associated L-functions is one of the central research areas in modern number theory, linking number theory, arithmetic geometry, representation theory, and complex analysis in many profound ways. The 19 papers cover a wide range of topics within the scope of the conference, including automorphic L-functions and their special values, p-adic modular forms, Eisenstein series, Borcherds products, automorphic periods, and many more.
Part 1 contains sections on Reductive groups, representations, Automorphic forms and

representations)

Many problems in number theory have simple statements, but their solutions require a deep understanding of algebra, algebraic geometry, complex analysis, group representations, or a combination of all four. The original simply stated problem can be obscured in the depth of the theory developed to understand it. This book is an introduction to some of these problems, and an overview of the theories used nowadays to attack them, presented so that the number theory is always at the forefront of the discussion. Lozano-Robledo gives an introductory survey of elliptic curves, modular forms, and L-functions. His main goal is to provide the reader with the big picture of the surprising connections among these three families of mathematical objects and their meaning for number theory. As a case in point, Lozano-Robledo explains the modularity theorem and its famous consequence, Fermat's Last Theorem. He also discusses the Birch and Swinnerton-Dyer Conjecture and other modern conjectures. The book begins with some motivating problems and includes numerous concrete examples throughout the text, often involving actual numbers, such as 3, 4, 5, $\frac{3344161}{747348}$, and $\frac{224403517704336969924557513090667486316094847204 1}{89123322689288595880255351789671635700164808 30}$. The theories of elliptic curves, modular forms, and L-functions are too vast to be covered in a single volume, and their proofs are outside the scope of the undergraduate curriculum. However, the

primary objects of study, the statements of the main theorems, and their corollaries are within the grasp of advanced undergraduates. This book concentrates on motivating the definitions, explaining the statements of the theorems and conjectures, making connections, and providing lots of examples, rather than dwelling on the hard proofs. The book succeeds if, after reading the text, students feel compelled to study elliptic curves and modular forms in all their glory.

Within the Langlands program, endoscopy is a fundamental process for relating automorphic representations of one group with those of another. In this book, Arthur establishes an endoscopic classification of automorphic representations of orthogonal and symplectic groups . The representations are shown to occur in families (known as global -packets and -packets), which are parametrized by certain self-dual automorphic representations of an associated general linear group . The central result is a simple and explicit formula for the multiplicity in the automorphic discrete spectrum of for any representation in a family. The results of the volume have already had significant applications: to the local Langlands correspondence, the construction of unitary representations, the existence of Whittaker models, the analytic behaviour of Langlands -functions, the spectral theory of certain locally symmetric spaces, and to new phenomena for symplectic epsilon-factors. One can expect many more. In fact, it is likely that both the results and the techniques of the volume will have applications to almost all sides of the Langlands program. The methods are by comparison of the trace

formula of with its stabilization (and a comparison of the twisted trace formula of with its stabilization, which is part of work in progress by Moeglin and Waldspurger). This approach is quite different from methods that are based on -functions, converse theorems, or the theta correspondence. The comparison of trace formulas in the volume ought to be applicable to a much larger class of groups. Any extension at all will have further important implications for the Langlands program.

This invaluable volume collects papers written by many of the world's top experts on L-functions. It not only covers a wide range of topics from algebraic and analytic number theories, automorphic forms, to geometry and mathematical physics, but also treats the theory as a whole. The contributions reflect the latest, most advanced and most important aspects of L-functions. In particular, it contains Hida's lecture notes at the conference and at the Eigenvariety semester in Harvard University and Weng's detailed account of his works on high rank zeta functions and non-abelian L-functions.

This book offers basic theory on hypercomplex-analytic automorphic forms and functions for arithmetic subgroups of the Vahlen group in higher dimensional spaces. Vector- and Clifford algebra-valued Eisenstein and Poincaré series are constructed within this framework and a detailed description of their analytic and number theoretical properties is provided. It establishes explicit relationships to generalized variants of the Riemann zeta function and Dirichlet L-series, and introduces hypercomplex multiplication of lattices.

"Six Short Chapters on Automorphic Forms and L-functions" treats the period conjectures of Shimura and the moment conjecture. These conjectures are of central importance in

contemporary number theory, but have hitherto remained little discussed in expository form. The book is divided into six short and relatively independent chapters, each with its own theme, and presents a motivated and lively account of the main topics, providing professionals an overall view of the conjectures and providing researchers intending to specialize in the area a guide to the relevant literature. Ze-Li Dou and Qiao Zhang are both associate professors of Mathematics at Texas Christian University, USA.

This graduate-level textbook provides an elementary exposition of the theory of automorphic representations and L-functions for the general linear group in an adelic setting. The authors keep definitions to a minimum and repeat them when reintroduced so that the book is accessible from any entry point, and with no prior knowledge of representation theory. They also include concrete examples of both global and local representations of GL(n), and present their associated L-functions. The theory is developed from first principles for GL(1), then carefully extended to GL(2) with complete detailed proofs of key theorems. Several of the proofs are here presented for the first time, including Jacquet's simple and elegant proof of the tensor product theorem. Finally, the higher rank situation of GL(n) is given a detailed treatment. Containing numerous exercises, this book will motivate students and researchers to begin working in this fertile field of research.

Detailed exposition of automorphic representations and their relation to string theory, for mathematicians and theoretical physicists.

In the early years of the 1980s, while I was visiting the Institute for Ad vanced Study (lAS) at Princeton as a postdoctoral member, I got a fascinating view, studying congruence modulo a prime among elliptic modular forms, that an automorphic L-function of a given algebraic group

G should have a canon ical p-adic counterpart of several variables. I immediately decided to find out the reason behind this phenomenon and to develop the theory of ordinary p-adic automorphic forms, allocating 10 to 15 years from that point, putting off the intended arithmetic study of Shimura varieties via L-functions and Eisenstein series (for which I visited lAS). Although it took more than 15 years, we now know (at least conjecturally) the exact number of variables for a given G, and it has been shown that this is a universal phenomenon valid for holomorphic automorphic forms on Shimura varieties and also for more general (nonholomorphic) cohomological automorphic forms on automorphic manifolds (in a markedly different way). When I was asked to give a series of lectures in the Automorphic Semester in the year 2000 at the Emile Borel Center (Centre Emile Borel) at the Poincare Institute in Paris, I chose to give an exposition of the theory of p-adic (ordinary) families of such automorphic forms p-adic analytically de pending on their weights, and this book is the outgrowth of the lectures given there.

This book provides a comprehensive account of the crucial role automorphic L-functions play in number theory and in the Langlands program, especially the Langlands functoriality conjecture. There has been a recent major development in the Langlands functoriality conjecture by the use of automorphic L-functions, namely, by combining converse theorems of Cogdell and Piatetski-Shapiro with the Langlands-Shahidi method. This book provides a step-by-step introduction to these developments and explains how the Langlands functoriality conjecture implies solutions to several outstanding conjectures in number theory, such as the Ramanujan conjecture, Sato-Tate conjecture, and Artin's conjecture. It would be ideal for an introductory course in the Langlands program. Titles in this series are co-published with The

Fields Institute for Research in Mathematical Sciences (Toronto, Ontario, Canada). Table of Contents: James W.Cogdell, Lectures on L-functions, converse theorems, and functoriality for GL_n: Preface; Modular forms and their L-functions; Automorphic forms; Automorphic representations; Fourier expansions and multiplicity one theorems; Eulerian integral representations; Local L-functions: The non-Archimedean case; The unramified calculation; Local L-functions: The Archimedean case; Global L-functions; Converse theorems; Functoriality; Functoriality for the classical groups; Functoriality for the classical groups, II. Henry H.Kim, Automorphic L-functions: Introduction; Chevalley groups and their properties; Cuspidal representations; L-groups and automorphic L-functions; Induced representations; Eisenstein series and constant terms; L-functions in the constant terms; Meromorphic continuation of L-functions; Generic representations and their Whittaker models; Local coefficients and non-constant terms; Local Langlands correspondence; Local L-functions and functional equations; Normalization of intertwining operators; Holomorphy and bounded in vertical strips; Langlands functoriality conjecture; Converse theorem of Cogdell and Piatetski-Shapiro; Functoriality of the symmetric cube; Functoriality of the symmetric fourth; Bibliography. M.Ram Murty, Applications of symmetric power L-functions: Preface; The Sato-Tate conjecture; Maass wave forms; The Rankin-Selberg method; Oscillations of Fourier coefficients of cusp forms; Poincare series; Kloosterman sums and Selberg's conjecture; Refined estimates for Fourier coefficients of cusp forms; Twisting and averaging of L-series; The Kim-Sarnak theorem; Introduction to Artin L-functions; Zeros and poles of Artin L-functions; The Langlands-Tunnell theorem; Bibliography. This is a reprint of the 2004 original. (FIM/20.S)

This volume contains the notes originally made by Kenkichi Iwasawa in his own handwriting for his lecture course at Princeton University in 1964. These notes give a beautiful and completely detailed account of the adelic approach to Hecke's L-functions attached to any number field, including the proof of analytic continuation, the functional equation of these L-functions, and the class number formula arising from the Dedekind zeta function for a general number field. This adelic approach was discovered independently by Iwasawa and Tate around 1950 and marked the beginning of the whole modern adelic approach to automorphic forms and L-series. While Tate's thesis at Princeton in 1950 was finally published in 1967 in the volume Algebraic Number Theory, edited by Cassels and Frohlich, no detailed account of Iwasawa's work has been published until now, and this volume is intended to fill the gap in the literature of one of the key areas of modern number theory. In the final chapter, Iwasawa elegantly explains some important classical results, such as the distribution of prime ideals and the class number formulae for cyclotomic fields.

An international Summer School on: "Modular functions of one variable and arithmetical applications" took place at RUCA, Antwerp University, from July 17 to - gust 3, 1972. This book is the first volume (in a series of four) of the Proceedings of the Summer School. It includes the basic course given by A. Ogg, and several other papers with a strong analyt~c flavour. Volume 2 contains the courses of R. P. Langlands (l-adic rep resentations) and P. Deligne (modular schemes - representations of GL) and papers on related topics. 2 Volume 3 is devoted to p-adic properties of modular forms and applications to l-adic representations and zeta functions. Volume 4 collects various material on elliptic curves, includ ing numerical tables. The School was a NATO Advanced Study Institute, and the orga nizers want to thank NATO for its major

subvention. Further support, in various forms, was received from IBM Belgium, the Coca-Cola Co. of Belgium, Rank Xerox Belgium, the Fort Food Co. of Belgium, and NSF Washington, D.C•• We extend our warm est thanks to all of them, as well as to RUCA and the local staff (not forgetting hostesses and secretaries!) who did such an excellent job.

The theory of automorphic forms has seen dramatic developments in recent years. In particular, important instances of Langlands functoriality have been established. This volume presents three weeks of lectures from the IAS/Park City Mathematics Institute Summer School on automorphic forms and their applications. It addresses some of the general aspects of automorphic forms, as well as certain recent advances in the field. The book starts with the lectures of Borel on the basic theory of automorphic forms, which lay the foundation for the lectures by Cogdell and Shahidi on converse theorems and the Langlands-Shahidi method, as well as those by Clozel and Li on the Ramanujan conjectures and graphs. The analytic theory of GL(2)-forms and L-functions are the subject of Michel's lectures, while Terras covers arithmetic quantum chaos. The volume also includes a chapter by Vogan on isolated unitary representations, which is related to the lectures by Clozel. This volume is recommended for independent study or an advanced topics course. It is suitable for graduate students and researchers interested in automorphic forms and number theory. the Institute for Advanced Study/Park City Mathematics Institute. Members of the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) receive a 20% discount from list price.

The main purpose of the book is to present the reader with various perspectives of the theory of automorphic forms. In addition to detailed and often nonstandard exposition of familiar topics

of the theory, with a particular emphasis on analytic aspects, special attention is paid to such subjects as theta-functions and representations of integers by quadratic forms.

Multiple Dirichlet Series, L-functions and Automorphic Forms gives the latest advances in the rapidly developing subject of Multiple Dirichlet Series, an area with origins in the theory of automorphic forms that exhibits surprising and deep connections to crystal graphs and mathematical physics. As such, it represents a new way in which areas including number theory, combinatorics, statistical mechanics, and quantum groups are seen to fit together. The volume also includes papers on automorphic forms and L-functions and related number-theoretic topics. This volume will be a valuable resource for graduate students and researchers in number theory, combinatorics, representation theory, mathematical physics, and special functions. Contributors: J. Beineke, B. Brubaker, D. Bump, G. Chinta, G. Cornelissen, C.A. Diaconu, S. Frechette, S. Friedberg, P. Garrett, D. Goldfeld, P.E. Gunnells, B. Heim, J. Hundley, D. Ivanov, Y. Komori, A.V. Kontorovich, O. Lorscheid, K. Matsumoto, P.J. McNamara, S.J. Patterson, M. Suzuki, H. Tsumura.

This volume uses a unified approach to representation theory and automorphic forms. It collects papers, written by leading mathematicians, that track recent progress in the expanding fields of representation theory and automorphic forms and their association with number theory and differential geometry. Topics include: Automorphic forms and distributions, modular forms, visible-actions, Dirac cohomology, holomorphic forms, harmonic analysis, self-dual representations, and Langlands Functoriality Conjecture, Both graduate students and researchers will find inspiration in this volume.

Since the pioneering work of Euler, Dirichlet, and Riemann, the analytic properties of L-

functions have been used to study the distribution of prime numbers. With the advent of the Langlands Program, L-functions have assumed a greater role in the study of the interplay between Diophantine questions about primes and representation theoretic properties of Galois representations. The present book provides a complete introduction to the most significant class of L-functions: the Artin-Hecke L-functions associated to finite-dimensional representations of Weil groups and to automorphic L-functions of principal type on the general linear group. In addition to establishing functional equations, growth estimates, and non-vanishing theorems, a thorough presentation of the explicit formulas of Riemann type in the context of Artin-Hecke and automorphic L-functions is also given. The survey is aimed at mathematicians and graduate students who want to learn about the modern analytic theory of L-functions and their applications in number theory and in the theory of automorphic representations. The requirements for a profitable study of this monograph are a knowledge of basic number theory and the rudiments of abstract harmonic analysis on locally compact abelian groups.

Part 2 contains sections on Automorphic representations and L-functions, Arithmetical algebraic geometry and L-functions

This book is the first of two volumes, which represent leading themes of current research in automorphic forms and representation theory of reductive groups over local fields. Articles in this volume mainly represent global aspects of automorphic forms. Among the topics are the trace formula; functoriality; representations of reductive groups over local fields; the relative trace formula and periods of automorphic forms;

Rankin - Selberg convolutions and L-functions; and, p-adic L-functions. The articles are written by leading researchers in the field, and bring the reader, advanced graduate students and researchers alike, to the frontline of the vigorous research in these deep, vital topics. The companion volume (""Contemporary Mathematics, Volume 489"") is devoted to local aspects of automorphic forms.

Aimed at presenting nontechnical explanations, all the essays in this collection of papers from the 1989 LMS Durham Symposium on L-functions are the contributions of renowned algebraic number theory specialists.

Part one of a two-volume collection exploring recent developments in number theory related to automorphic forms and Galois representations.

This volume is the proceedings of the conference on Automorphic Representations, L-functions and Applications: Progress and Prospects, held at the Department of Mathematics of The Ohio State University, March 27–30, 2003, in honor of the 60th birthday of Steve Rallis. The theory of automorphic representations, automorphic L-functions and their applications to arithmetic continues to be an area of vigorous and fruitful research. The contributed papers in this volume represent many of the most recent developments and directions, including Rankin–Selberg L-functions (Bump, Ginzburg–Jiang–Rallis, Lapid–Rallis) the relative trace formula (Jacquet, Mao–Rallis) automorphic representations (Gan–Gurevich, Ginzburg–Rallis–Soudry) representation theory of p-adic groups (Baruch, Kudla–Rallis, Mœglin, Cogdell–Piatetski-

Shapiro–Shahidi) p-adic methods (Harris–Li–Skinner, Vigneras), and arithmetic applications (Chinta–Friedberg–Hoffstein). The survey articles by Bump, on the Rankin–Selberg method, and by Jacquet, on the relative trace formula, should be particularly useful as an introduction to the key ideas about these important topics. This volume should be of interest both to researchers and students in the area of automorphic representations, as well as to mathematicians in other areas interested in having an overview of current developments in this important field.

These notes are concerned with showing the relation between L-functions of classical groups (*F1 in particular) and *F2 functions arising from the oscillator representation of the dual reductive pair *F1 *F3 O(Q). The problem of measuring the nonvanishing of a *F2 correspondence by computing the Petersson inner product of a *F2 lift from *F1 to O(Q) is considered. This product can be expressed as the special value of an L-function (associated to the standard representation of the L-group of *F1) times a finite number of local Euler factors (measuring whether a given local representation occurs in a given oscillator representation). The key ideas used in proving this are (i) new Rankin integral representations of standard L-functions, (ii) see-saw dual reductive pairs and (iii) Siegel-Weil formula. The book addresses readers who specialize in the theory of automorphic forms and L-functions and the representation theory of Lie groups. N

This modern, graduate-level textbook does not assume prior knowledge of representation theory. Includes numerous concrete examples and over 250 exercises.

Automorphic forms are an important complex analytic tool in number theory and modern arithmetic geometry. They played for example a vital role in Andrew Wiles's proof of Fermat's Last Theorem. This text provides a concise introduction to the world of automorphic forms using two approaches: the classic elementary theory and the modern point of view of adeles and representation theory. The reader will learn the important aims and results of the theory by focussing on its essential aspects and restricting it to the 'base field' of rational numbers. Students interested for example in arithmetic geometry or number theory will find that this book provides an optimal and easily accessible introduction into this topic.

This graduate-level textbook provides an elementary exposition of the theory of automorphic representations and L-functions for the general linear group in an adelic setting. Definitions are kept to a minimum and repeated when reintroduced so that the book is accessible from any entry point, and with no prior knowledge of representation theory. The book includes concrete examples of global and local representations of GL(n), and presents their associated L-functions. In Volume 1, the theory is developed from first principles for GL(1), then carefully extended to GL(2) with complete detailed proofs of key theorems. Several proofs are presented for the first time, including Jacquet's simple and elegant proof of the tensor product theorem. In Volume 2, the higher rank situation of GL(n) is

given a detailed treatment. Containing numerous exercises by Xander Faber, this book will motivate students and researchers to begin working in this fertile field of research.

James W. Cogdell, Lectures on L-functions, converse theorems, and functoriality for GL_n: Preface Modular forms and their L-functions Automorphic forms Automorphic representations Fourier expansions and multiplicity one theorems Eulerian integral representations Local L-functions: The non-Archimedean case The unramified calculation Local L-functions: The Archimedean case Global L-functions Converse theorems Functoriality Functoriality for the classical groups Functoriality for the classical groups, II Henry H. Kim, Automorphic L-functions: Introduction Chevalley groups and their properties Cuspidal representations L-groups and automorphic L-functions Induced representations Eisenstein series and constant terms L-functions in the constant terms Meromorphic continuation of L-functions Generic representations and their Whittaker models Local coefficients and non-constant terms Local Langlands correspondence Local L-functions and functional equations Normalization of intertwining operators Holomorphy and bounded in vertical strips Langlands functoriality conjecture Converse theorem of Cogdell and Piatetski-Shapiro Functoriality of the symmetric cube Functoriality of

the symmetric fourth Bibliography M. Ram Murty, Applications of symmetric power L-functions: Preface The Sato-Tate conjecture Maass wave forms The Rankin-Selberg method Oscillations of Fourier coefficients of cusp forms Poincare series Kloosterman sums and Selberg's conjecture Refined estimates for Fourier coefficients of cusp forms Twisting and averaging of L-series The Kim-Sarnak theorem Introduction to Artin L-functions Zeros and poles of Artin L-functions The Langlands-Tunnell theorem Bibliography

L-functions associated to automorphic forms encode all classical number theoretic information. They are akin to elementary particles in physics. This 2006 book provides an entirely self-contained introduction to the theory of L-functions in a style accessible to graduate students with a basic knowledge of classical analysis, complex variable theory, and algebra. Also within the volume are many new results not yet found in the literature. The exposition provides complete detailed proofs of results in an easy-to-read format using many examples and without the need to know and remember many complex definitions. The main themes of the book are first worked out for GL(2,R) and GL(3,R), and then for the general case of GL(n,R). In an appendix to the book, a set of Mathematica functions is presented, designed to allow the reader to explore the theory from a computational point of view.

This book is the second of two volumes, which represent leading themes of current research in automorphic forms and representation theory of reductive groups over local fields. Articles in this volume mainly represent global aspects of automorphic forms. Among the topics are the trace formula; functoriality; representations of reductive groups over local fields; the relative trace formula and periods of automorphic forms; Rankin - Selberg convolutions and L-functions; and, p-adic L-functions. The articles are written by leading researchers in the field, and bring the reader, advanced graduate students and researchers alike, to the frontline of the vigorous research in these deep, vital topics. The companion volume (""Contemporary Mathematics, Volume 488"") is devoted to global aspects of automorphic forms.

This book discusses the p-adic modular forms, the eigencurve that parameterize them, and the p-adic L-functions one can associate to them. These theories and their generalizations to automorphic forms for group of higher ranks are of fundamental importance in number theory. For graduate students and newcomers to this field, the book provides a solid introduction to this highly active area of research. For experts, it will offer the convenience of collecting into one place foundational definitions and theorems with complete and self-contained proofs. Written in an engaging and educational style, the book also includes

exercises and provides their solution.