An Introduction To Mathematical Optimal Control Theory

This book strives to provide a balanced coverage of efficient algorithms commonly used in solving mathematical optimization problems. It covers both the convectional algorithms and modern heuristic and metaheuristic methods. Topics include gradient-based algorithms such as Newton-Raphson method, steepest descent method, Hooke-Jeeves pattern search, Lagrange multipliers, linear programming, particle swarm optimization (PSO), simulated annealing (SA), and Tabu search. Multiobjective optimization including important concepts such as Pareto optimality and utility method is also described. Three Matlab and Octave programs so as to demonstrate how PSO and SA work are provided. An example of demonstrating how to modify these programs to solve multiobjective optimization problems using recursive method is discussed.

This is an intuitively motivated presentation of many topics in classical mechanics and related areas of control theory and calculus of variations. All topics throughout the book are treated with zero tolerance for unrevealing definitions and for proofs which leave the reader in the dark. Some areas of particular interest are: an extremely short derivation of the ellipticity of planetary orbits; a statement and an explanation of the "tennis racket paradox"; a heuristic explanation (and a rigorous treatment) of the gyroscopic effect; a revealing equivalence between the dynamics of a particle and statics of a spring; a short geometrical explanation of Pontryagin's Maximum Principle, and more. In the last chapter, aimed at more advanced readers, the Hamiltonian and the momentum are compared to forces in a certain static problem. This gives a palpable physical meaning to some seemingly abstract concepts and theorems. With minimal prerequisites consisting of basic calculus and basic undergraduate physics, this book is suitable for courses from an undergraduate to a beginning graduate level, and for a mixed audience of mathematics, physics and engineering students. Much of the enjoyment of the subject lies in solving almost 200 problems in this book.

This book showcases a subclass of hereditary systems, that is, systems with behaviour depending not only on their current state but also on their past history; it is an introduction to the mathematical theory of optimal control for stochastic difference Volterra equations of neutral type. As such, it will be of much interest to researchers interested in modelling processes in physics, mechanics, automatic regulation, economics and finance, biology, sociology and medicine for all of which such equations are very popular tools. The text deals with problems of optimal control such as meeting given performance criteria, and stabilization, extending them to neutral stochastic difference Volterra equations. In particular, it contrasts the difference analogues of solutions to optimal control and optimal estimation problems for stochastic integral Volterra equations with optimal solutions for corresponding problems in stochastic difference Volterra equations. Optimal

Control of Stochastic Difference Volterra Equations commences with an historical introduction to the emergence of this type of equation with some additional mathematical preliminaries. It then deals with the necessary conditions for optimality in the control of the equations and constructs a feedback control scheme. The approximation of stochastic quasilinear Volterra equations with quadratic performance functionals is then considered. Optimal stabilization is discussed and the filtering problem formulated. Finally, two methods of solving the optimal control problem for partly observable linear stochastic processes, also with quadratic performance functionals, are developed. Integrating the author’s own research within the context of the current state-of-the-art of research in difference equations, hereditary systems theory and optimal control, this book is addressed to specialists in mathematical optimal control theory and to graduate students in pure and applied mathematics and control engineering.

This text covers the basic theory and computation for a first course in linear programming, including substantial material on mathematical proof techniques and sophisticated computation methods. Includes Appendix on using Excel. 1984 edition.

Solving optimization problems subject to constraints given in terms of partial d- ferential equations (PDEs) with additional constraints on the controls and/or states is one of the most challenging problems in the context of industrial, medical and economical applications, where the transition from model-based numerical si- lations to model-based design and optimal control is crucial. For the treatment of such optimization problems the interaction of optimization techniques and num- ical simulation plays a central role. After proper discretization, the number of op- 3 10 timization variables varies between 10 and 10 . It is only very recently that the enormous advances in computing power have made it possible to attack problems of this size. However, in order to accomplish this task it is crucial to utilize and f- ther explore the speci?c mathematical structure of optimization problems with PDE constraints, and to develop new mathematical approaches concerning mathematical analysis, structure exploiting algorithms, and discretization, with a special focus on prototype applications. The present book provides a modern introduction to the rapidly developing ma- ematical ?eld of optimization with PDE constraints. The ?rst chapter introduces to the analytical background and optimality theory for optimization problems with PDEs. Optimization problems with PDE-constraints are posed in in?nite dim- sional spaces. Therefore, functional analytic techniques, function space theory, as well as existence- and uniqueness results for the underlying PDE are essential to study the existence of optimal solutions and to derive optimality conditions.

This monograph is an introduction to optimal control theory for systems governed by vector ordinary differential equations. It is not intended as a state-of-the-art handbook for researchers. We have tried to keep two types of reader in mind: (1) mathematicians, graduate students, and advanced undergraduates in mathematics who want a concise introduction to a field which contains nontrivial interesting applications of mathematics (for example, weak convergence, convexity, and the theory of ordinary differential equations); (2) economists, applied scientists, and engineers who want to understand some of the mathematical foundations. of optimal control theory. In general, we have emphasized motivation and explanation, avoiding the "definition-axiom-theorem-proof" approach. We make use of a large number of examples, especially one simple canonical example which we carry through the entire book. In proving theorems, we often just prove the simplest case, then state the more general results which can be proved. Many of the more difficult topics are discussed in the "Notes" sections at the end of chapters and several major proofs are in the Appendices. We feel that a solid understanding of basic facts is best attained by at first avoiding excessive generality. We have not tried to give an exhaustive list of references, preferring to refer the reader to existing books or papers with extensive bibliographies. References are given by author's name and the year of publication, e.g., Waltman [1974].

This work familiarises students with mathematical models (PDEs) and methods of numerical solution and optimisation. Including numerous exercises and examples, this is an ideal text for advanced students in Applied Mathematics, Engineering, Physical Science and Computer Science.

This book presents basic optimization principles and gradient-based algorithms to a general audience, in a brief and easy-to-read form. It enables professionals to apply optimization theory to engineering, physics, chemistry, or business economics.

Conceived by Count Jacopo Francesco Riccati more than a quarter of a millennium ago, the Riccati equation has been widely studied in the subsequent centuries. Since its introduction in control theory in the sixties, the matrix Riccati equation has known an impressive range of applications, such as optimal control, H? optimization and robust stabilization, stochastic realization, synthesis of linear passive networks, to name but a few. This book consists of 11 chapters surveying the main concepts and results related to the matrix Riccati equation, both in continuous and discrete time. Theory, applications and numerical algorithms are extensively presented in an expository way. As a foreword, the history and prehistory of the Riccati equation is concisely presented.

Mathematical Control Theory: An Introduction presents, in a mathematically precise manner, a unified introduction to deterministic control theory. In addition to classical concepts and ideas, the author covers the stabilization of nonlinear systems using topological methods, realization theory for nonlinear systems, impulsive control and positive systems, the control of rigid bodies, the stabilization of infinite dimensional systems, and the solution of minimum energy problems.

"Covers a remarkable number of topics....The book presents a large amount of material very well, and its use is highly recommended." --Bulletin of the AMS

Numerous examples highlight this treatment of the use of linear quadratic Gaussian methods for control system design. It explores linear optimal control theory from an engineering viewpoint, with illustrations of practical applications. Key topics include loop-recovery techniques, frequency shaping, and controller reduction. Numerous examples and complete solutions. 1990 edition.

Upper-level undergraduate text introduces aspects of optimal control theory: dynamic programming, Pontryagin's minimum principle, and numerical techniques for trajectory optimization. Numerous figures, tables. Solution guide available upon request. 1970 edition.

This self-contained beginning graduate text covers linear and integer programming, polytopes, matroids and matroid optimization, shortest paths, and network flows.

This book should be considered as an introduction to a special dass of hierarchical systems of optimal control, where subsystems are described by partial differential equations of various types. Optimization is carried out by means of a two-level scheme, where the center optimizes coordination for the upper level and subsystems find the optimal solutions for independent local problems. The main algorithm is a method of iterative aggregation. The coordinator solves the problern with macrovariables, whose number is less than the number of initial variables. This problern is often very simple. On the lower level, we have the usual optimal control problems of math ematical physics, which are far simpler than the initial statements. Thus, the decomposition (or reduction to problems ofless dimensions) is obtained. The algorithm constructs a sequence of so-called disaggregated solutions that are feasible for the main problern and converge to its optimal solutionunder certain assumptions (e.g., under strict convexity of the input functions). Thus, we bridge the gap between two disciplines: optimization theory of large-scale systems and mathematical physics. The first motivation was a special model of branch planning, where the final product obeys a preset assortment relation. The ratio coefficient is maximized. Constraints are given in the form of linear inequalities with block diagonal structure of the part of a matrix that corresponds to subsystems. The central coordinator assem bles the final production from the components produced by the subsystems.

This book provides an introduction to the theory of linear systems and control for students in business mathematics, econometrics, computer science, and engineering; the focus is on discrete time systems. The subjects treated are among the central topics of deterministic linear system theory: controllability, observability, realization theory, stability and stabilization by feedback, LQ-optimal control theory. Kalman filtering and LQC-control of stochastic systems are also discussed, as are modeling, time series analysis and model specification, along with model validation.

Mathematical Optimization Terminology: A Comprehensive Glossary of Terms is a practical book with the essential formulations, illustrative examples, real-world applications and main references on the topic. This book helps readers gain a more practical understanding of optimization, enabling them to apply it to their algorithms. This book also addresses the need for a practical publication that introduces these concepts and techniques. Discusses real-world applications of optimization and how it can be used in algorithms Explains the essential formulations of optimization in mathematics Covers a more practical approach to optimization

The performance of a process -- for example, how an aircraft consumes fuel -- can be enhanced when the most effective controls and operating points for the process are determined. This holds true for many physical, economic, biomedical, manufacturing, and engineering processes whose behavior can often be influenced by altering certain parameters or controls to optimize some desired property or output.

A comprehensive introduction to the tools, techniques and applications of convex optimization.

The calculus of variations is used to find functions that optimize quantities expressed in terms of integrals. Optimal control theory seeks to find functions that minimize cost integrals for systems described by differential equations. This book is an introduction to both the classical theory of the calculus of variations and the more modern developments of optimal control theory from the perspective of an applied mathematician. It focuses on understanding concepts and how to apply them. The range of potential applications is broad: the calculus of variations and optimal control theory have been widely used in numerous ways in biology, criminology, economics, engineering, finance, management science, and physics. Applications described in this book include cancer chemotherapy, navigational control, and renewable resource harvesting. The prerequisites for the book are modest: the standard calculus sequence, a first course on ordinary differential equations, and some facility with the use of mathematical software. It is suitable for an undergraduate or beginning graduate course, or for self study. It provides excellent preparation for more advanced books and courses on the calculus of variations and optimal control theory.

Mathematical programming: an overview; solving linear programs; sensitivity analysis; duality in linear programming; mathematical programming in practice; integration of strategic and tactical planning in the aluminum industry; planning the mission and composition of the U.S. merchant Marine fleet; network models; integer programming; design of a naval tender job shop; dynamic programming; large-scale systems; nonlinear programming; a system for bank portfolio planning; vectors and matrices; linear programming in matrix form; a labeling algorithm for the maximun-flow network problem.

This textbook offers a concise yet rigorous introduction to calculus of variations and optimal control theory, and is a self-contained resource for graduate students in engineering, applied mathematics, and related subjects. Designed specifically for a one-semester course, the book begins with calculus of variations, preparing the ground for optimal control. It then gives a complete proof of the maximum principle and covers key topics such as the Hamilton-Jacobi-Bellman theory of dynamic programming and linear-quadratic optimal control. Calculus of Variations and Optimal Control Theory also traces the historical development of the subject and features numerous exercises, notes and references at the end of each chapter, and suggestions for further study. Offers a concise yet rigorous introduction Requires limited background in control theory or advanced mathematics Provides a complete proof of the maximum principle Uses consistent notation in the exposition of classical and modern topics Traces the historical development of the subject Solutions manual (available only to teachers) Leading universities that have adopted this book include: University of Illinois at Urbana-Champaign ECE 553: Optimum Control Systems Georgia Institute of Technology ECE 6553: Optimal Control and Optimization University of Pennsylvania ESE 680: Optimal Control Theory University of Notre Dame EE 60565: Optimal Control

A modern, up-to-date introduction to optimization theory andmethods This authoritative book serves as an introductory text tooptimization at the senior undergraduate and beginning graduatelevels. With consistently accessible and elementary treatment ofall topics, An Introduction to Optimization, Second Edition helpsstudents build a solid working knowledge of the field, includingunconstrained optimization, linear programming, and constrainedoptimization. Supplemented with more than one hundred tables and illustrations,an extensive bibliography, and numerous worked examples toillustrate both theory and algorithms, this book alsoprovides: * A review of the required mathematical background material * A mathematical discussion at a level accessible to MBA andbusiness students * A treatment of both linear and nonlinear programming * An introduction to recent developments, including neuralnetworks, genetic algorithms, and interior-point methods * A chapter on the use of descent algorithms for the training offeedforward neural networks * Exercise problems after every chapter, many new to thisedition * MATLAB(r) exercises and examples * Accompanying Instructor's Solutions Manual available onrequest An Introduction to Optimization, Second Edition helps studentsprepare for the advanced topics and technological developments thatlie ahead. It is also a useful book for researchers andprofessionals in mathematics, electrical engineering, economics,statistics, and business. An Instructor's Manual presenting detailed solutions to all theproblems in the book is available from the Wiley editorialdepartment.

This textbook is a straightforward introduction to the theory of optimal control with an emphasis on presenting many different applications. Professor Hocking has taken pains to ensure that the theory is developed to display the main themes of the arguments but without using sophisticated mathematical tools. Throughout there are many worked examples, and numerous exercises (with solutions) are provided.

Combining control theory and modeling, this textbook introduces and builds on methods for simulating and tackling concrete problems in a variety of applied sciences. Emphasizing "learning by doing," the authors focus on examples and applications to real-world problems. An elementary presentation of advanced concepts, proofs to introduce new ideas, and carefully presented MATLAB® programs help foster an understanding of the basics, but also lead the way to new, independent research. With minimal prerequisites and exercises in each chapter, this work serves as an excellent textbook and reference for graduate and advanced undergraduate students, researchers, and practitioners in mathematics, physics, engineering, computer science, as well as biology, biotechnology, economics, and finance.

Geared primarily to an audience consisting of mathematically advanced undergraduate or beginning graduate students, this text may additionally be used by engineering students interested in a rigorous, proof-oriented systems course that goes beyond the classical frequency-domain material and more applied courses. The minimal mathematical background required is a working knowledge of linear algebra and differential equations. The book covers what constitutes the common core of control theory and is unique in its emphasis on foundational aspects. While covering a wide range of topics written in a standard theorem/proof style, it also develops the necessary techniques from scratch. In this second edition, new chapters and sections have been added, dealing with time optimal control of linear systems, variational and numerical approaches to nonlinear control, nonlinear controllability via Lie-algebraic methods, and controllability of recurrent nets and of linear systems with bounded controls.

Introduction to Mathematical OptimizationFrom Linear Programming to MetaheuristicsCambridge International Science Pub

A fun and stunningly illustrated introduction to the art of linear optimization Linear optimization is a powerful modeling method for discovering the best solution to a problem among a set of available alternatives. It is one of today’s most important branches of mathematics and computer science—and also a surprisingly rich medium for creating breathtaking works of art. Opt Art takes readers on an entertaining tour of linear optimization and its applications, showing along the way how it can be used to design visual art. Robert Bosch provides a lively and accessible introduction to the geometric, algebraic, and algorithmic foundations of optimization. He presents classical applications, such as the legendary Traveling Salesman Problem, and shows how to adapt them to make optimization art—opt art. Each chapter in this marvelously illustrated book begins with a problem or puzzle and demonstrates how the solution can be derived using a host of artistic methods and media, including 3D printing, laser cutting, and computer-controlled machining. Bosch focuses on mathematical modeling throughout—converting a problem into a workable mathematical form, solving it using optimization techniques, and examining the results, which can take the form of mosaics, line drawings, and even sculpture. All you need is some high-school algebra, geometry, and calculus to follow along. Featuring more than a hundred illustrations and photos of Bosch’s own art, Opt Art demonstrates how mathematics and computing can be used to create beauty and express emotion through amazing works of art.

This treatment focuses on the analysis and algebra underlying the workings of convexity and duality and necessary/sufficient local/global optimality conditions for unconstrained and constrained optimization problems. 2015 edition.

"Optimal control theory is concerned with finding control functions that minimize cost functions for systems described by differential equations. The methods have found widespread applications in aeronautics, mechanical engineering, the life sciences, and many other disciplines. This book focuses on optimal control problems where the state equation is an elliptic or parabolic partial differential equation. Included are topics such as the existence of optimal solutions, necessary optimality conditions and adjoint equations, second-order sufficient conditions, and main principles of selected numerical techniques. It also contains a survey on the Karush-Kuhn-Tucker theory of nonlinear programming in Banach spaces. The exposition begins with control problems with linear equations, quadratic cost functions and control constraints. To make the book self-contained, basic facts on weak solutions of elliptic and parabolic equations are introduced. Principles of functional analysis are introduced and explained as they are needed. Many simple examples illustrate the theory and its hidden difficulties. This start to the book makes it fairly self-contained and suitable for advanced undergraduates or beginning graduate students. Advanced control problems for nonlinear partial differential equations are also discussed. As prerequisites, results on boundedness and continuity of solutions to semilinear elliptic and parabolic equations are addressed. These topics are not yet readily available in books on PDEs, making the exposition also interesting for researchers. Alongside the main theme of the analysis of problems of optimal control, Tr'oltzsch also discusses numerical techniques. The exposition is confined to brief introductions into the basic ideas in order to give the reader an impression of how the theory can be realized numerically. After reading this book, the reader will be familiar with the main principles of the numerical analysis of PDE-constrained optimization."--Publisher's description.

This book contains extended, in-depth presentations of the plenary talks from the 16th French-German-Polish Conference on Optimization, held in Kraków, Poland in 2013. Each chapter in this book exhibits a comprehensive look at new theoretical and/or application-oriented results in mathematical modeling, optimization, and optimal control. Students and researchers involved in image processing, partial differential inclusions, shape optimization, or optimal control theory and its applications to medical and rehabilitation technology, will find this book valuable. The first chapter by Martin Burger provides an overview of recent developments related to Bregman distances, which is an important tool in inverse problems and image processing. The chapter by Piotr Kalita studies the operator version of a first order in time partial differential inclusion and its time discretization. In the chapter by Günter Leugering, Jan Soko?owski and Antoni ?ochowski, nonsmooth shape optimization problems for variational inequalities are considered. The next chapter, by Katja Mombaur is devoted to applications of optimal control and inverse optimal control in the field of medical and rehabilitation technology, in particular in human movement analysis, therapy and improvement by means of medical devices. The final chapter, by Nikolai Osmolovskii and Helmut Maurer provides a survey on no-gap second order optimality conditions in the calculus of variations and optimal control, and a discussion of their further development.

Optimization is an essential technique for solving problems in areas as diverse as accounting, computer science and engineering. Assuming only basic linear algebra and with a clear focus on the fundamental concepts, this textbook is the perfect starting point for first- and second-year undergraduate students from a wide range of backgrounds and with varying levels of ability. Modern, real-world examples motivate the theory throughout. The authors keep the text as concise and focused as possible, with more advanced material treated separately or in starred exercises. Chapters are self-contained so that instructors and students can adapt the material to suit their own needs and a wide selection of over 140 exercises gives readers the opportunity to try out the skills they gain in each section. Solutions are available for instructors. The book also provides suggestions for further reading to help students take the next step to more advanced material.

This book serves as an introductory text in mathematical programming and optimization for students having a mathematical background that includes one semester of linear algebra and a complete calculus sequence. It includes computational examples to aid students develop computational skills.

When the Tyrian princess Dido landed on the North African shore of the Mediterranean sea she was welcomed by a local chieftain. He offered her all the land that she could enclose between the shoreline and a rope of knotted cowhide. While the legend does not tell us, we may assume that Princess Dido arrived at the correct solution by stretching the rope into the shape of a circular arc and thereby maximized the area of the land upon which she was to found Carthage. This story of the founding of Carthage is apocryphal. Nonetheless it is probably the first account of a problem of the kind that inspired an entire mathematical discipline, the calculus of variations and its extensions such as the theory of optimal control. This book is intended to present an introductory treatment of the calculus of variations in Part I and of optimal control theory in Part II. The discussion in Part I is restricted to the simplest problem of the calculus of variations. The topic is entirely classical; all of the basic theory had been developed before the turn of the century. Consequently the material comes from many sources; however, those most useful to me have been the books of Oskar Bolza and of George M. Ewing. Part II is devoted to the elementary aspects of the modern extension of the calculus of variations, the theory of optimal control of dynamical systems.

This book is intended to be a teaching aid for students of the courses in Operations Research and Mathematical Optimization for scientific faculties. Some of the basic topics of Operations Research and Optimization are considered: Linear Programming, Integer Linear Programming, Computational Complexity, and Graph Theory. Particular emphasis is given to Integer Linear Programming, with an exposition of the most recent resolution techniques, and in particular of the branch-and-cut method. The work is accompanied by numerous examples and exercises.

This book focuses on maximum principle and verification theorem for incomplete information forward-backward stochastic differential equations (FBSDEs) and their applications in linear-quadratic optimal controls and mathematical finance. ?Lots of interesting phenomena arising from the area of mathematical finance can be described by FBSDEs. Optimal control problems of FBSDEs are theoretically important and practically relevant. A standard assumption in the literature is that the stochastic noises in the model are completely observed. However, this is rarely the case in real world situations. The optimal control problems under complete information are studied extensively. Nevertheless, very little is known about these problems when the information is not complete. The aim of this book is to fill this gap. This book is written in a style suitable for graduate students and researchers in mathematics and engineering with basic knowledge of stochastic process, optimal control and mathematical finance.