An Introduction To Lebesgue Integration And Fourier Series

A uniquely accessible book for general measure and integration, emphasizing the real line, Euclidean space, and the underlying role of translation in real analysis Measure and Integration: A Concise Introduction to Real Analysis presents the basic concepts and methods that are important for successfully reading and understanding proofs. Blending coverage of both fundamental and specialized topics, this book serves as a practical and thorough introduction to measure and integration, while also facilitating a basic understanding of real analysis. The author develops the theory of measure and integration on abstract measure spaces with an emphasis of the real line and Euclidean space. Additional topical coverage includes: Measure spaces, outer measures, and extension theorems Lebesgue measure on the line and in Euclidean space Measurable functions, Egoroff's theorem, and Lusin's theorem Convergence theorems for integrals Product measures and Fubini's theorem Differentiation theorems for functions of real variables Decomposition theorems for signed measures Absolute continuity and the Radon-Nikodym theorem Lp spaces, continuous-function spaces, and duality theorems Translation-invariant subspaces of L2 and applications The book's presentation lays the foundation for further study of functional analysis, harmonic analysis, and probability, and its treatment of real analysis highlights the fundamental role of translations. Each theorem is accompanied by opportunities to employ the concept, as numerous exercises explore applications including convolutions, Fourier transforms, and differentiation across the integral sign. Providing an efficient and readable treatment of this classical subject, Measure and Integration: A Concise Introduction to Real Analysis is a useful book for courses in real analysis at the graduate level. It is also a valuable reference for practitioners in the mathematical sciences.

An Introduction to Lebesgue Integration and Fourier SeriesCourier Corporation

Undergraduate-level introduction to Riemann integral, measurable sets, measurable functions, Lebesgue integral, other topics. Numerous examples and exercises.

In 1902, modern function theory began when Henri Lebesgue described a new "integral calculus." His "Lebesgue integral" handles more functions than the traditional integral-so many more that mathematicians can study collections (spaces) of functions. For example, it defines a distance between any two functions in a space. This book describes these ideas in an elementary accessible way. Anyone who has mastered calculus concepts of limits, derivatives, and series can enjoy the material. Unlike any other text, this book brings analysis research topics within reach of readers even just beginning to think about functions from a theoretical point of view.

In this book, Hawkins elegantly places Lebesgue's early work on integration theory within in proper historical context by relating it to the developments during the nineteenth century that motivated it and gave it significance and also to the contributions made in this field by Lebesgue's contemporaries. Hawkins was awarded the 1997 MAA Chauvenet Prize and the 2001 AMS Albert Leon Whiteman Memorial Prize for notable exposition and exceptional scholarship in the history of mathematics.

A User-Friendly Introduction to Lebesgue Measure and Integration provides a bridge between an undergraduate course in Real Analysis and a first graduate-level course in Measure Theory and Integration. The main goal of this book is to prepare students for what they may encounter in graduate school, but will be useful for many beginning graduate students as well. The book starts with the fundamentals of measure theory that are gently approached through the very concrete example of Lebesgue measure. With this approach, Lebesgue integration becomes a natural extension of Riemann integration. Next, -spaces are defined. Then the book turns to a discussion of limits, the basic idea covered in a first analysis course. The book also discusses in detail such questions as: When does a sequence of Lebesgue integrable functions converge to a Lebesgue integrable function? What does that say about the sequence of integrals? Another core idea from a first analysis course is completeness. Are these -spaces complete? What exactly does that mean in this setting? This book concludes with a brief overview of General Measures. An appendix contains suggested projects suitable for end-of-course papers or presentations. The book is written in a very reader-friendly manner, which makes it appropriate for students of varying degrees of preparation, and the only prerequisite is an undergraduate course in Real Analysis.

This book covers the material of a one year course in real analysis. It includes an original axiomatic approach to Lebesgue integration which the authors have found to be effective in the classroom. Each chapter contains numerous examples and an extensive problem set which expands considerably the breadth of the material covered in the text. Hints are included for some of the more difficult problems.

While mathematics students generally meet the Riemann integral early in their undergraduate studies, those whose interests lie more in the direction of applied mathematics will probably find themselves needing to use the Lebesgue or Lebesgue-Stieltjes Integral before they have acquired the necessary theoretical background. This book is aimed at exactly this group of readers. The authors introduce the Lebesgue-Stieltjes integral on the real line as a natural extension of the Riemann integral, making the treatment as practical as possible. They discuss the evaluation of Lebesgue-Stieltjes integrals in detail, as well as the standard convergence theorems, and conclude with a brief discussion of multivariate integrals and surveys of L spaces plus some applications. The whole is rounded off with exercises that extend and illustrate the theory, as well as providing practice in the techniques.

This textbook, based on three series of lectures held by the author at the University of Strasbourg, presents functional analysis in a non-traditional way by generalizing elementary theorems of plane geometry to spaces of arbitrary dimension. This approach leads naturally to the basic notions and theorems. Most results are illustrated by the small lp spaces. The Lebesgue integral, meanwhile, is treated via the direct approach of Frigyes Riesz, whose constructive definition of measurable functions leads to optimal, clear-cut versions of the classical theorems of Fubini-Tonelli and Radon-Nikodým. Lectures on Functional Analysis and the Lebesgue Integral presents the most important topics for students, with short, elegant proofs. The exposition style follows the Hungarian mathematical tradition of Paul Erd?s and others. The order of the first two parts, functional analysis and the Lebesgue integral, may be reversed. In the third and final part they are combined to study various spaces of continuous and integrable functions. Several beautiful, but almost forgotten, classical theorems are also included. Both undergraduate and graduate students in pure and applied mathematics, physics and engineering will find this textbook useful. Only basic topological notions and results are used and various simple but pertinent examples and exercises illustrate the usefulness and optimality of most theorems. Many of these examples are new or difficult to localize in the literature, and the original sources of most notions and results are indicated to help the reader understand the genesis and development of the field.

This book presents a compact and self-contained introduction to the theory of measure and integration. The introduction into this theory is as necessary (because of its multiple applications) as difficult for the uninitiated. Most measure theory treaties involve a large amount of prerequisites and present crucial theoretical challenges. By taking on another approach, this textbook provides less experienced readers with material that allows an easy access to the definition and main properties of the Lebesgue integral. The book will be welcomed by upper undergraduate/early graduate students who wish to better understand certain concepts and results of probability theory, statistics, economic equilibrium theory, game theory, etc., where the Lebesgue integral makes its presence felt throughout. The book can also be useful to students in the faculties of mathematics, physics, computer science, engineering, life sciences, as an introduction to a more in-depth study of measure theory.

Meant for advanced undergraduate and graduate students in mathematics, this introduction to measure theory and Lebesgue integration is motivated by the historical questions that led to its development. The author tells the story of the mathematicians who wrestled with the difficulties inherent in the Riemann integral, leading to the work of Jordan, Borel, and Lebesgue.

Consists of two separate but closely related parts. Originally published in 1966, the first section deals with elements of integration and has been updated and corrected. The latter half details the main concepts of Lebesgue measure and uses the abstract measure space approach of the Lebesgue integral because it strikes directly at the most important results—the convergence theorems.

This text approaches integration via measure theory as opposed to measure theory via integration, an approach which makes it easier to grasp the subject. Apart from its central importance to pure mathematics, the material is also relevant to applied mathematics and probability, with proof of the mathematics set out clearly and in considerable detail. Numerous worked examples necessary for teaching and learning at undergraduate level constitute a strong feature of the book, and after studying statements of results of the theorems, students should be able to attempt the 300 problem exercises which test comprehension and for which detailed solutions are provided. Approaches integration via measure theory, as opposed to measure theory via integration, making it easier to understand the subject Includes numerous worked examples necessary for teaching and learning at undergraduate level Detailed solutions are provided for the 300 problem exercises which test comprehension of the theorems provided

This concise introduction to Lebesgue integration is geared toward advanced undergraduate math majors and may be read by any student possessing some familiarity with real variable theory and elementary calculus. The self-contained treatment features exercises at the end of each chapter that range from simple to difficult. The approach begins with sets and functions and advances to Lebesgue measure, including considerations of measurable sets, sets of measure zero, and Borel sets and nonmeasurable sets. A two-part exploration of the integral covers measurable functions, convergence theorems, convergence in mean, Fourier theory, and other topics. A chapter on calculus examines change of variables, differentiation of integrals, and integration of derivatives and by parts. The text concludes with a consideration of more general measures, including absolute continuity and convolution products. Dover (2014) republication of the edition originally published by Holt, Rinehart & Winston, New York, 1962. See every Dover book in print at www.doverpublications.com

This book provides a student's first encounter with the concepts of measure theory and functional analysis. Its structure and content reflect the belief that difficult concepts should be introduced in their simplest and most concrete forms. Despite the use of the word ``terse'' in the title, this text might also have been called A (Gentle) Introduction to Lebesgue Integration. It is terse in the sense that it treats only a subset of those concepts typically found in a substantial graduate-level analysis course. The book emphasizes the motivation of these concepts and attempts to treat them simply and concretely. In particular, little mention is made of general measures other than Lebesgue until the final chapter and attention is limited to R as opposed to R^n. After establishing the primary ideas and results, the text moves on to some applications. Chapter 6 discusses classical real and complex Fourier series for L^2 functions on the interval and shows that the Fourier series of an L^2 function converges in L^2 to that function. Chapter 7 introduces some concepts from measurable dynamics. The Birkhoff ergodic theorem is stated without proof and results on Fourier series from Chapter 6 are used to prove that an irrational rotation of the circle is ergodic and that the squaring map on the complex numbers of modulus 1 is ergodic. This book is suitable for an advanced undergraduate course or for the start of a graduate course. The text presupposes that the student has had a standard undergraduate course in real analysis.

This is a graduate text introducing the fundamentals of measure theory and integration theory, which is the foundation of modern real analysis. The text focuses first on the concrete setting of Lebesgue measure and the Lebesgue integral (which in turn is motivated by the more classical concepts of Jordan measure and the Riemann integral), before moving on to abstract measure and integration theory, including the standard convergence theorems, Fubini's theorem, and the Carathéodory extension theorem. Classical differentiation theorems, such as the Lebesgue and Rademacher differentiation theorems, are also covered, as are connections with probability theory. The material is intended to cover a quarter or semester's worth of material for a first graduate course in real analysis. There is an emphasis in the text on tying together the abstract and the concrete sides of the subject, using the latter to illustrate and motivate the former. The central role of key principles (such as Littlewood's three principles) as providing guiding intuition to the subject is also emphasized. There are a large number of exercises throughout that develop key aspects of the theory, and are thus an integral component of the text. As a supplementary section, a discussion of general problem-solving strategies in analysis is also given. The last three sections discuss optional topics related to the main matter of the book.

This paperback, gives a self-contained treatment of the theory of finite measures in general spaces at the undergraduate level.

"'Lebesgue Integration on Euclidean Space' contains a concrete, intuitive, and patient derivation of Lebesgue measure and integration on Rn. It contains many exercises that are incorporated throughout the text, enabling the reader to apply immediately the new ideas that have been presented" --

This book presents a historical development of the integration theories of Riemann, Lebesgue, Henstock-Kurzweil, and McShane, showing how new theories of integration were developed to solve problems that earlier theories could not handle. It develops the basic properties of each integral in detail and provides comparisons of the different integrals. The chapters covering each integral are essentially independent and can be used separately in teaching a portion of an introductory course on real analysis. There is a sufficient supply of exercises to make the book useful as a textbook.

These well-known and concise lecture notes present the fundamentals of the Lebesgue theory of integration and an introduction to some of the theory's applications. Suitable for advanced undergraduates and graduate students of mathematics, the treatment also covers topics of interest to practicing analysts. Author Harold Widom emphasizes the construction and properties of measures in general and Lebesgue measure in particular as well as the definition of the integral and its main properties. The notes contain chapters on the Lebesgue spaces and their duals, differentiation of measures in Euclidean space, and the application of integration theory to Fourier series.

Aimed primarily at undergraduate level university students, An Illustrative Introduction to Modern Analysis provides an accessible and lucid contemporary account of the fundamental principles of Mathematical Analysis. The themes treated include Metric Spaces, General Topology, Continuity, Completeness, Compactness, Measure Theory, Integration, Lebesgue Spaces, Hilbert Spaces, Banach Spaces, Linear Operators, Weak and Weak* Topologies. Suitable both for classroom use and independent reading, this book is ideal preparation for further study in research areas where a broad mathematical toolbox is required.

The theory of integration is one of the twin pillars on which analysis is built. The first version of integration that students see is the Riemann integral. Later, graduate students learn that the Lebesgue integral is ``better'' because it removes some restrictions on the integrands and the domains over which we integrate. However, there are still drawbacks to Lebesgue integration, for instance, dealing with the Fundamental Theorem of Calculus, or with ``improper'' integrals. This book is an introduction to a relatively new theory of the integral (called the ``generalized Riemann integral'' or the ``Henstock-Kurzweil integral'') that corrects the defects in the classical Riemann theory and both simplifies and extends the Lebesgue theory of integration. Although this integral includes that of Lebesgue, its definition is very close to the Riemann integral that is familiar to students from calculus. One virtue of the new approach is that no measure theory and virtually no topology is required. Indeed, the book includes a study of measure theory as an application of the integral. Part 1 fully develops the theory of the integral of functions defined on a compact interval. This restriction on the domain is not necessary, but it is the case of most interest and does not exhibit some of the technical problems that can impede the reader's understanding. Part 2 shows how this theory extends to functions defined on the whole real line. The theory of Lebesgue measure from the integral is then developed, and the author makes a connection with some of the traditional approaches to the Lebesgue integral. Thus, readers are given full exposure to the main classical results. The text is suitable for a first-year graduate course, although much of it can be readily mastered by advanced undergraduate students. Included are many examples and a very rich collection of exercises. There are partial solutions to approximately one-third of the exercises. A complete solutions manual is available separately.

This book covers Lebesgue integration and its generalizations from Daniell's point of view, modified by the use of seminorms. Integrating functions rather than measuring sets is posited as the main purpose of measure theory. From this point of view Lebesgue's integral can be had as a rather straightforward, even simplistic, extension of Riemann's integral; and its aims, definitions, and procedures can be motivated at an elementary level. The notion of measurability, for example, is suggested by Littlewood's observations rather than being conveyed authoritatively through definitions of (sigma)-algebras and good-cut-conditions, the latter of which are hard to justify and thus appear mysterious, even nettlesome, to the beginner. The approach taken provides the additional benefit of cutting the labor in half. The use of seminorms, ubiquitous in modern analysis, speeds things up even further. The book is intended for the reader who has some experience with proofs, a beginning graduate student for example. It might even be useful to the advanced mathematician who is confronted with situations - such as stochastic integration - where the set-measuring approach to integration does not work.

Responses from colleagues and students concerning the first edition indicate that the text still answers a pedagogical need which is not addressed by other texts. There are no major changes in this edition. Several proofs have been tightened, and the exposition has been modified in minor ways for improved clarity. As before, the strength of the text lies in presenting the student with the difficulties which led to the development of the theory and, whenever possi ble, giving the student the tools to overcome those difficulties for himself or herself. Another proverb: Give me a fish, I eat for a day. Teach me to fish, I eat for a lifetime. Soo Bong Chae March 1994 Preface to the First Edition This book was developed from lectures in a course at New College and should be accessible to advanced undergraduate and beginning graduate students. The prerequisites are an understanding of introductory calculus and the ability to comprehend "e-I) arguments. " The study of abstract measure and integration theory has been in vogue for more than two decades in American universities since the publication of Measure Theory by P. R. Halmos (1950). There are, however, very few ele mentary texts from which the interested reader with a calculus background can learn the underlying theory in a form that immediately lends itself to an understanding of the subject. This book is meant to be on a level between calculus and abstract integration theory for students of mathematics and physics.

This is a sequel to Dr Weir's undergraduate textbook on Lebesgue Integration and Measure (CUP. 1973) in which he provided a concrete approach to the Lebesgue integral in terms of step functions and went on from there to deduce the abstract concept of Lebesgue measure. In this second volume, the treatment of the Lebesgue integral is generalised to give the Daniell integral and the related general theory of measure. This approach via integration of elementary functions is particularly well adapted to the proof of Riesz's famous theorems about linear functionals on the classical spaces C (X) and LP and also to the study of topological notions such as Borel measure. This book will be used for final year honours courses in pure mathematics and for graduate courses in functional analysis and measure theory.

Dr Burkill gives a straightforward introduction to Lebesgue's theory of integration. His approach is the classical one, making use of the concept of measure, and deriving the principal results required for applications of the theory.

This very well written and accessible book emphasizes the reasons for studying measure theory, which is the foundation of much of probability. By focusing on measure, many illustrative examples and applications, including a thorough discussion of standard probability distributions and densities, are opened. The book also includes many problems and their fully worked solutions.

This book describes integration and measure theory for readers interested in analysis, engineering, and economics. It gives a systematic account of Riemann-Stieltjes integration and deduces the Lebesgue-Stieltjes measure from the Lebesgue-Stieltjes integral.

This open access textbook welcomes students into the fundamental theory of measure, integration, and real analysis. Focusing on an accessible approach, Axler lays the foundations for further study by promoting a deep understanding of key results. Content is carefully curated to suit a single course, or two-semester sequence of courses, creating a versatile entry point for graduate studies in all areas of pure and applied mathematics. Motivated by a brief review of Riemann integration and its deficiencies, the text begins by immersing students in the concepts of measure and integration. Lebesgue measure and abstract measures are developed together, with each providing key insight into the main ideas of the other approach. Lebesgue integration links into results such as the Lebesgue Differentiation Theorem. The development of products of abstract measures leads to Lebesgue measure on Rn. Chapters on Banach spaces, Lp spaces, and Hilbert spaces showcase major results such as the Hahn–Banach Theorem, Hölder’s Inequality, and the Riesz Representation Theorem. An in-depth study of linear maps on Hilbert spaces culminates in the Spectral Theorem and Singular Value Decomposition for compact operators, with an optional interlude in real and complex measures. Building on the Hilbert space material, a chapter on Fourier analysis provides an invaluable introduction to Fourier series and the Fourier transform. The final chapter offers a taste of probability. Extensively class tested at multiple universities and written by an award-winning mathematical expositor, Measure, Integration & Real Analysis is an ideal resource for students at the start of their journey into graduate mathematics. A prerequisite of elementary undergraduate real analysis is assumed; students and instructors looking to reinforce these ideas will appreciate the electronic Supplement for Measure, Integration & Real Analysis that is freely available online.

Unified Integration

Elementary Introduction to the Lebesgue Integral is not just an excellent primer of the Lebesgue integral for undergraduate students but a valuable tool for tomorrow’s mathematicians. Since the early twentieth century, the Lebesgue integral has been a mainstay of mathematical analysis because of its important properties with respect to limits. For this reason, it is vital that mathematical students properly understand the complexities of the Lebesgue integral. However, most texts about the subject are geared towards graduate students, which makes it a challenge for instructors to properly teach and for less advanced students to learn. Ensuring that the subject is accessible for all readers, the author presents the text in a clear and concrete manner which allows readers to focus on the real line. This is important because Lebesgue integral can be challenging to understand when compared to more widely used integrals like the Riemann integral. The author also includes in the textbook abundant examples and exercises to help explain the topic. Other topics explored in greater detail are abstract measure spaces and product measures, which are treated concretely. Features: Comprehensibly written introduction to the Lebesgue integral for undergraduate students Includes many examples, figures and exercises Features a Table of Notation and Glossary to aid readers Solutions to selected exercises

The Lebesgue integral is now standard for both applications and advanced mathematics. This books starts with a review of the familiar calculus integral and then constructs the Lebesgue integral from the ground up using the same ideas. A Primer of Lebesgue Integration has been used successfully both in the classroom and for individual study. Bear presents a clear and simple introduction for those intent on further study in higher mathematics. Additionally, this book serves as a refresher providing new insight for those in the field. The author writes with an engaging, commonsense style that appeals to readers at all levels.

Real Analysis: Measures, Integrals and Applications is devoted to the basics of integration theory and its related topics. The main emphasis is made on the properties of the Lebesgue integral and various applications both classical and those rarely covered in literature. This book provides a detailed introduction to Lebesgue measure and integration as well as the classical results concerning integrals of multivariable functions. It examines the concept of the Hausdorff measure, the properties of the area on smooth and Lipschitz surfaces, the divergence formula, and Laplace's method for finding the asymptotic behavior of integrals. The general theory is then applied to harmonic analysis, geometry, and topology. Preliminaries are provided on probability theory, including the study of the Rademacher functions as a sequence of independent random variables. The book contains more than 600 examples and exercises. The reader who has mastered the first third of the book will be able to study other areas of mathematics that use integration, such as probability theory, statistics, functional analysis, partial probability theory, statistics, functional analysis, partial differential equations and others. Real Analysis: Measures, Integrals and Applications is intended for advanced undergraduate and graduate students in mathematics and physics. It assumes that the reader is familiar with basic linear algebra and differential calculus of functions of several variables.

The Lebesgue integral is the standard tool for mathematics, physics, or engineering. It is used where sequences or series arise and where one might wish to express the integral of a sum of integrals or vice versa. Itis essential in Fourier analysis and for the construction of Hilbert space and other function spaces. This primer gives a concrete treatment, building Lebesgue theory in a way parallel to the Riemann integral of beginning calculus. It makes the powerfultools of Lebesgue theory readily available to those in applied areas. It is a good introduction for those going on in mathematics as well as a refresher holding new insights for those in the field. Professor Bear gives a clear and engaging account of this important and previously forbidding piece of mathematics, providing readers with a key to many areas of analysis. * * The Riemann-Darboux Integral * The Riemann Integral as a Limit of Sums * Lebesgue Measure on (0.1) * Measurable-Sets - The Caratheodory Characterization

This volume develops the classical theory of the Lebesgue integral and some of its applications. The integral is initially presented in the context of n-dimensional Euclidean space, following a thorough study of the concepts of outer measure and measure. A more general treatment of the integral, based on an axiomatic approach, is later given. Closely related topics in real variables, such as functions of bounded variation, the Riemann-Stieltjes integral, Fubini's theorem, L(p)) classes, and various results about differentiation are examined in detail. Several applications of the theory to a specific branch of analysis--harmonic analysis--are also provided. Among these applications are basic facts about convolution operators and Fourier series, including results for the conjugate function and the Hardy-Littlewood maximal function. Measure and Integral: An Introduction to Real Analysis provides an introduction to real analysis for student interested in mathematics, statistics, or probability. Requiring only a basic familiarity with advanced calculus, this volume is an excellent textbook for advanced undergraduate or first-year graduate student in these areas.

Real Analysis is the third volume in the Princeton Lectures in Analysis, a series of four textbooks that aim to present, in an integrated manner, the core areas of analysis. Here the focus is on the development of measure and integration theory, differentiation and integration, Hilbert spaces, and Hausdorff measure and fractals. This book reflects the objective of the series as a whole: to make plain the organic unity that exists between the various parts of the subject, and to illustrate the wide applicability of ideas of analysis to other fields of mathematics and science. After setting forth the basic facts of measure theory, Lebesgue integration, and differentiation on Euclidian spaces, the authors move to the elements of Hilbert space, via the L2 theory. They next present basic illustrations of these concepts from Fourier analysis, partial differential equations, and complex analysis. The final part of the book introduces the reader to the fascinating subject of fractional-dimensional sets, including Hausdorff measure, self-replicating sets, space-filling curves, and Besicovitch sets. Each chapter has a series of exercises, from the relatively easy to the more complex, that are tied directly to the text. A substantial number of hints encourage the reader to take on even the more challenging exercises. As with the other volumes in the series, Real Analysis is accessible to students interested in such diverse disciplines as mathematics, physics, engineering, and finance, at both the undergraduate and graduate levels. Also available, the first two volumes in the Princeton Lectures in Analysis:

A textbook for the undergraduate who is meeting the Lebesgue integral for the first time, relating it to the calculus and exploring its properties before deducing the consequent notions of measurable functions and measure.

A superb text on the fundamentals of Lebesgue measure and integration. This book is designed to give the reader a solid understanding of Lebesgue measure and integration. It focuses on only the most fundamental concepts, namely Lebesgue measure for R and Lebesgue integration for extended real-valued functions on R. Starting with a thorough presentation of the preliminary concepts of undergraduate analysis, this book covers all the important topics, including measure theory, measurable functions, and integration. It offers an abundance of support materials, including helpful illustrations, examples, and problems. To further enhance the learning experience, the author provides a historical context that traces the struggle to define "area" and "area under a curve" that led eventually to Lebesgue measure and integration. Lebesgue Measure and Integration is the ideal text for an advanced undergraduate analysis course or for a first-year graduate course in mathematics, statistics, probability, and other applied areas. It will also serve well as a supplement to courses in advanced measure theory and integration and as an invaluable reference long after course work has been completed.

This self-contained treatment of measure and integration begins with a brief review of the Riemann integral and proceeds to a construction of Lebesgue measure on the real line. From there the reader is led to the general notion of measure, to the construction of the Lebesgue integral on a measure space, and to the major limit theorems, such as the Monotone and Dominated Convergence Theorems. The treatment proceeds to L^p spaces, normed linear spaces that are shown to be complete (i.e., Banach spaces) due to the limit theorems. Particular attention is paid to L^2 spaces as Hilbert spaces, with a useful geometrical structure. Having gotten quickly to the heart of the matter, the text proceeds to broaden its scope. There are further constructions of measures, including Lebesgue measure on n-dimensional Euclidean space. There are also discussions of surface measure, and more generally of Riemannian manifolds and the measures they inherit, and an appendix on the integration of differential forms. Further geometric aspects are explored in a chapter on Hausdorff measure. The text also treats probabilistic concepts, in chapters on ergodic theory, probability spaces and random variables, Wiener measure and Brownian motion, and martingales. This text will prepare graduate students for more advanced studies in functional analysis, harmonic analysis, stochastic analysis, and geometric measure theory.