

## Algebraic Geometry Imperial College London

Despite the renown of the Fields Medals, J.C. Fields has been until now a rather obscure figure, and recovering details about his professional activities and personal life was not at all a simple task. This work is a triumph of persistence with far-flung archival and documentary sources, and provides a rich non-mathematical portrait of the man in all aspects of his life and career. Highly readable and replete with period detail, the book sheds useful light on the mathematical and scientific world of Fields' time, and is sure to remain the definitive biographical study. --Tom Archibald, Simon Fraser University, Burnaby, BC, Canada Drawing on a wide array of archival sources, Riehm and Hoffman provide a vivid account of Fields' life and his part in the founding of the highest award in mathematics. Filled with intriguing detail--from a childhood on the shores of Lake Ontario, through the mathematics seminars of late 19th century Berlin, to the post-WW1 years of the fragmented international mathematical community--it is a richly textured story engagingly and sympathetically told. Read this book and you will understand why Fields never wanted the medal to bear his name and yet why, quite rightly, it does. --June Barrow-Green, Open University, Milton Keynes, United Kingdom One of the little-known effects of World War I was the collapse of international scientific cooperation. In mathematics, the discord continued after the war's end and after the Treaty of Versailles had been signed in 1919. Many distinguished scientists were involved in the war and its aftermath, and from their letters and papers, now almost a hundred years old, we learn of their anguished wartime views and their struggles afterwards either to prolong the schism in mathematics or to end it. J.C. Fields, the foremost Canadian mathematician of his time, was educated in Canada, the United States, and Germany, and championed an international spirit of cooperation to further the frontiers of mathematics. It was during the awkward post-war period that J.C. Fields established the Fields Medal, an international prize for outstanding research, which soon became the highest award in mathematics. J.C. Fields intended it to be an international medal, and a glance at the varying backgrounds of the fifty-two Fields medallists shows it to be so. Who was Fields? What carried him from Hamilton, Canada West, where he was born in 1863, into the middle of this turbulent era of international scientific politics? A modest mathematician, he was an unassuming man. This biography outlines Fields' life and times and the difficult circumstances in which he created the Fields Medal. It is the first such published study.

The field of geometry reflects a conglomeration of discoveries over time. Filled with detailed diagrams, this insightful volume offers serious students a comprehensive understanding of the fundamentals of geometry, including geometric shapes, axioms, and formulas. In addition, it covers some of the field's most illustrious minds, from Euclid to Wendelin Werner, figures who have helped produce the various branches of geometry as we know them today. This enlightening volume will help students understand the principles of geometry, and also the fascinating story behind the numbers.

Spine title: AISMC-3 : artificial intelligence and symbolic mathematical computation.

"Analytic and algebraic geometers often study the same geometric structures but bring different methods to bear on them. While this dual approach has been spectacularly successful at solving problems, the language differences between algebra and analysis also represent a difficulty for students and researchers in geometry, particularly complex geometry. The PCMI program was designed to partially address this language gulf, by presenting some of the active developments in algebraic and analytic geometry in a form suitable for students on the 'other side' of the analysis-algebra language divide. One focal point of the summer school was multiplier ideals, a subject of wide current interest in both subjects. The present volume is based on a series of lectures at the PCMI summer school on analytic and algebraic geometry. The series is designed to give a high-level introduction to the advanced techniques behind some recent developments in algebraic and analytic geometry. The lectures contain many illustrative examples, detailed computations, and new perspectives on the topics presented, in order to enhance access of this material to non-specialists."--Publisher's description.

The series is designed to bring together those mathematicians who are seriously interested in getting new challenging stimuli from economic theories with those economists who are seeking effective mathematical tools for their research. A lot of economic problems can be formulated as constrained optimizations and equilibration of their solutions. Various mathematical theories have been supplying economists with indispensable machineries for these problems arising in economic theory. Conversely, mathematicians have been stimulated by various mathematical difficulties raised by economic theories.

Moduli theory is the study of how objects, typically in algebraic geometry but sometimes in other areas of mathematics, vary in families and is fundamental to an understanding of the objects themselves. First formalised in the 1960s, it represents a significant topic of modern mathematical research with strong connections to many areas of mathematics (including geometry, topology and number theory) and other disciplines such as theoretical physics. This book, which arose from a programme at the Isaac Newton Institute in Cambridge, is an ideal way for graduate students and more experienced researchers to become acquainted with the wealth of ideas and problems in moduli theory and related areas. The reader will find articles on both fundamental material and cutting-edge research topics, such as: algebraic stacks; BPS states and the  $P = W$  conjecture; stability conditions; derived differential geometry; and counting curves in algebraic varieties, all written by leading experts.

This book concerns the theory of unipotent elements in simple algebraic groups over algebraically closed or finite fields, and nilpotent elements in the corresponding simple Lie algebras. These topics have been an important area of study for decades, with applications to representation theory, character theory, the subgroup structure of algebraic groups and finite groups, and the classification of the finite simple groups. The main focus is on obtaining full information on class representatives and centralizers of unipotent and nilpotent elements. Although there is a substantial literature on this topic, this book is the first single source where such information is presented completely in all characteristics. In addition, many of the results are new--for example, those concerning centralizers of nilpotent elements in small characteristics. Indeed, the whole approach, while using some ideas from the literature, is novel, and yields many new general and specific facts concerning the structure and embeddings of centralizers. A comprehensive collection of expository articles on cutting-edge topics at the forefront of research in algebraic geometry.

The fundamental mathematical tools needed to understand machine learning include linear algebra, analytic geometry, matrix decompositions, vector calculus, optimization, probability and statistics. These topics are traditionally taught in disparate courses, making it hard for data science or computer science students, or professionals, to efficiently learn the mathematics. This self-contained textbook bridges the gap between mathematical and machine learning texts, introducing the mathematical concepts with a minimum of prerequisites. It uses these concepts to derive four central machine learning methods: linear regression, principal component analysis, Gaussian mixture models and support vector machines. For students and others with a mathematical background, these derivations provide a starting point to

machine learning texts. For those learning the mathematics for the first time, the methods help build intuition and practical experience with applying mathematical concepts. Every chapter includes worked examples and exercises to test understanding. Programming tutorials are offered on the book's web site.

This monograph provides a systematic treatment of the Brauer group of schemes, from the foundational work of Grothendieck to recent applications in arithmetic and algebraic geometry. The importance of the cohomological Brauer group for applications to Diophantine equations and algebraic geometry was discovered soon after this group was introduced by Grothendieck. The Brauer–Manin obstruction plays a crucial role in the study of rational points on varieties over global fields. The birational invariance of the Brauer group was recently used in a novel way to establish the irrationality of many new classes of algebraic varieties. The book covers the vast theory underpinning these and other applications. Intended as an introduction to cohomological methods in algebraic geometry, most of the book is accessible to readers with a knowledge of algebra, algebraic geometry and algebraic number theory at graduate level. Much of the more advanced material is not readily available in book form elsewhere; notably, de Jong's proof of Gabber's theorem, the specialisation method and applications of the Brauer group to rationality questions, an in-depth study of the Brauer–Manin obstruction, and proof of the finiteness theorem for the Brauer group of abelian varieties and K3 surfaces over finitely generated fields. The book surveys recent work but also gives detailed proofs of basic theorems, maintaining a balance between general theory and concrete examples. Over half a century after Grothendieck's foundational seminars on the topic, *The Brauer–Grothendieck Group* is a treatise that fills a longstanding gap in the literature, providing researchers, including research students, with a valuable reference on a central object of algebraic and arithmetic geometry.

Torsors, also known as principal bundles or principal homogeneous spaces, are ubiquitous in mathematics. The purpose of this book is to present expository lecture notes and cutting-edge research papers on the theory and applications of torsors and étale homotopy, all written from different perspectives by leading experts. Part one of the book contains lecture notes on recent uses of torsors in geometric invariant theory and representation theory, plus an introduction to the étale homotopy theory of Artin and Mazur. Part two of the book features a milestone paper on the étale homotopy approach to the arithmetic of rational points. Furthermore, the reader will find a collection of research articles on algebraic groups and homogeneous spaces, rational and K3 surfaces, geometric invariant theory, rational points, descent and the Brauer–Manin obstruction. Together, these give a state-of-the-art view of a broad area at the crossroads of number theory and algebraic geometry.

This volume contains the proceedings of two AMS Special Sessions "Geometric and Algebraic Aspects of Representation Theory" and "Quantum Groups and Noncommutative Algebraic Geometry" held October 13–14, 2012, at Tulane University, New Orleans, Louisiana. Included in this volume are original research and some survey articles on various aspects of representations of algebras including Kac–Moody algebras, Lie superalgebras, quantum groups, toroidal algebras, Leibniz algebras and their connections with other areas of mathematics and mathematical physics.

This book constitutes the refereed proceedings of the 6th Conference on Computability in Europe, CiE 2010, held in Ponta Delgada, Azores, Portugal, in June/July 2010. The 28 revised papers presented together with 20 invited lectures were carefully reviewed and selected from 90 submissions. The papers address not only the more established lines of research of computational complexity and the interplay between proofs and computation, but also novel views that rely on physical and biological processes and models to find new ways of tackling computations and improving their efficiency.

This development of the theory of complex algebraic curves was one of the peaks of nineteenth century mathematics. They have many fascinating properties and arise in various areas of mathematics, from number theory to theoretical physics, and are the subject of much research. By using only the basic techniques acquired in most undergraduate courses in mathematics, Dr. Kirwan introduces the theory, observes the algebraic and topological properties of complex algebraic curves, and shows how they are related to complex analysis. Many important applications in global optimization, algebra, probability and statistics, applied mathematics, control theory, financial mathematics, inverse problems, etc. can be modeled as a particular instance of the Generalized Moment Problem (GMP). This book introduces a new general methodology to solve the GMP when its data are polynomials and basic semi-algebraic sets. This methodology combines semidefinite programming with recent results from real algebraic geometry to provide a hierarchy of semidefinite relaxations converging to the desired optimal value. Applied on appropriate cones, standard duality in convex optimization nicely expresses the duality between moments and positive polynomials. In the second part, the methodology is particularized and described in detail for various applications, including global optimization, probability, optimal control, mathematical finance, multivariate integration, etc., and examples are provided for each particular application. Errata(s). Errata. Sample Chapter(s). Chapter 1: The Generalized Moment Problem (227 KB). Contents: Moments and Positive Polynomials: The Generalized Moment Problem; Positive Polynomials; Moments; Algorithms for Moment Problems; Applications: Global Optimization over Polynomials; Systems of Polynomial Equations; Applications in Probability; Markov Chains Applications; Application in Mathematical Finance; Application in Control; Convex Envelope and Representation of Convex Sets; Multivariate Integration; Min-Max Problems and Nash Equilibria; Bounds on Linear PDE. Readership: Postgraduates, academics and researchers in mathematical programming, control and optimization.

This book provides a comprehensive introduction to the interactions between noncommutative algebra and classical algebraic geometry. Offers information on entrance and degree requirements, expenses and financial aid, programs of study, and faculty research specialties.

Featuring a blend of original research papers and comprehensive surveys from an international team of leading researchers in the thriving fields of foliation theory, holomorphic foliations, and birational geometry, this book presents the proceedings of the conference "Foliation Theory in Algebraic Geometry," hosted by the Simons Foundation in New York City in September 2013. Topics covered include: Fano and del Pezzo foliations; the cone theorem and rank one foliations; the structure of symmetric differentials on a smooth complex surface and a local structure theorem for closed symmetric differentials of rank two; an overview of lifting symmetric differentials from varieties with canonical singularities and the applications to the classification of AT bundles on singular varieties; an overview of the powerful theory of the variety of minimal rational tangents introduced by Hwang and Mok; recent examples of varieties which are hyperbolic and yet the Green-Griffiths locus is the whole of  $X$ ; and a classification of pseudoeffective codimension one distributions. Foliations play a fundamental role in algebraic geometry, for example in the proof of abundance for threefolds and to a solution of the Green-Griffiths conjecture for surfaces of general type with positive Segre class. The purpose of this volume is to foster communication and enable interactions between experts who work on holomorphic foliations and birational geometry, and to bring together leading researchers to demonstrate the powerful connection of ideas, methods, and goals shared by these two areas of study.

This title introduces the theory of arc schemes in algebraic geometry and singularity theory, with special emphasis on recent developments around the Nash problem for surfaces. The main challenges are to understand the global and local structure of arc schemes, and how they relate to the nature of the singularities on the variety. Since the arc scheme is an infinite dimensional object, new tools need to be developed to give a precise meaning to the notion of a singular point of the arc scheme. Other related topics are also explored, including motivic integration and dual intersection complexes of resolutions of singularities. Written by

leading international experts, it offers a broad overview of different applications of arc schemes in algebraic geometry, singularity theory and representation theory.

This volume is based on lectures delivered at the 2019 AMS Short Course “Sum of Squares: Theory and Applications”, held January 14–15, 2019, in Baltimore, Maryland. This book provides a concise state-of-the-art overview of the theory and applications of polynomials that are sums of squares. This is an exciting and timely topic, with rich connections to many areas of mathematics, including polynomial and semidefinite optimization, real and convex algebraic geometry, and theoretical computer science. The six chapters introduce and survey recent developments in this area; specific topics include the algebraic and geometric aspects of sums of squares and spectrahedra, lifted representations of convex sets, and the algorithmic and computational implications of viewing sums of squares as a meta algorithm. The book also showcases practical applications of the techniques across a variety of areas, including control theory, statistics, finance and machine learning.

An up-to-date account of algebraic statistics and information geometry, which also explores the emerging connections between these two disciplines.

This book contains a collection of survey articles of exciting new developments in geometry, written in tribute to Simon Donaldson to celebrate his 60th birthday. Reflecting the wide range of Donaldson's interests and influence, the papers range from algebraic geometry and topology through symplectic geometry and geometric analysis to mathematical physics. Their expository nature means the book acts as an invitation to the various topics described, while also giving a sense of the links between these different areas and the unity of modern geometry.

The articles in this volume cover a broad range of topics in algebraic geometry: classical varieties, linear system, birational geometry, Minimal Model Program, moduli spaces, toric varieties, enumerative theory of singularities, equivariant cohomology and arithmetic questions.

This volume collects contributions written by different experts in honor of Prof. Jaime Muñoz Masqué. It covers a wide variety of research topics, from differential geometry to algebra, but particularly focuses on the geometric formulation of variational calculus; geometric mechanics and field theories; symmetries and conservation laws of differential equations, and pseudo-Riemannian geometry of homogeneous spaces. It also discusses algebraic applications to cryptography and number theory. It offers state-of-the-art contributions in the context of current research trends. The final result is a challenging panoramic view of connecting problems that initially appear distant.

This volume honors Sir Peter Swinnerton-Dyer's mathematical career spanning more than 60 years' of amazing creativity in number theory and algebraic geometry.

Now back in print, this highly regarded book has been updated to reflect recent advances in the theory of semistable coherent sheaves and their moduli spaces, which include moduli spaces in positive characteristic, moduli spaces of principal bundles and of complexes, Hilbert schemes of points on surfaces, derived categories of coherent sheaves, and moduli spaces of sheaves on Calabi–Yau threefolds. The authors review changes in the field since the publication of the original edition in 1997 and point the reader towards further literature. References have been brought up to date and errors removed. Developed from the authors' lectures, this book is ideal as a text for graduate students as well as a valuable resource for any mathematician with a background in algebraic geometry who wants to learn more about Grothendieck's approach.

This volume consists of the written presentations of lectures given at two special sessions: the AMS Special Session on Topology in Dynamics (Winston-Salem, NC) and the AMS-AWM Special Session on Geometry in Dynamics (San Antonio, TX). Each article concerns aspects of the topology or geometry of dynamical systems. Topics covered include the following: foliations and laminations, iterated function systems, the three-body problem, isotopy stability, homoclinic tangles, fractal dimension, Morse homology, knotted orbits, inverse limits, contact structures, Grassmanians, blowups, and continua. New results are presented reflecting current trends in topological aspects of dynamical systems. The book offers a wide variety of topics of special interest to those working this area bridging topology and dynamical systems.

The theory of Riemann surfaces occupies a very special place in mathematics. It is a culmination of much of traditional calculus, making surprising connections with geometry and arithmetic. It is an extremely useful part of mathematics, knowledge of which is needed by specialists in many other fields. It provides a model for a large number of more recent developments in areas including manifold topology, global analysis, algebraic geometry, Riemannian geometry, and diverse topics in mathematical physics. This graduate text on Riemann surface theory proves the fundamental analytical results on the existence of meromorphic functions and the Uniformisation Theorem. The approach taken emphasises PDE methods, applicable more generally in global analysis. The connection with geometric topology, and in particular the role of the mapping class group, is also explained. To this end, some more sophisticated topics have been included, compared with traditional texts at this level. While the treatment is novel, the roots of the subject in traditional calculus and complex analysis are kept well in mind. Part I sets up the interplay between complex analysis and topology, with the latter treated informally. Part II works as a rapid first course in Riemann surface theory, including elliptic curves. The core of the book is contained in Part III, where the fundamental analytical results are proved. Following this section, the remainder of the text illustrates various facets of the more advanced theory.

More than a study of shapes and angles, geometry reflects an amalgamation of discoveries over time. This book not only provides readers with a comprehensive understanding of geometric shapes, axioms, and formulas, it presents the field's brilliant minds—from Euclid to Wendelin Werner and many in between—whose works reflect a progression of mathematical thought throughout the centuries and have helped produce the various branches of geometry as they are known today. Detailed diagrams illustrate various concepts and help make geometry accessible to all.

An accessible introduction to convex algebraic geometry and semidefinite optimization. For graduate students and researchers in mathematics and computer science.

This is Part 1 of a two-volume set. Since Oscar Zariski organized a meeting in 1954, there has been a major algebraic geometry meeting every decade: Woods Hole (1964), Arcata (1974), Bowdoin (1985), Santa Cruz (1995), and Seattle (2005). The American Mathematical Society has supported these summer institutes for over 50 years. Their proceedings volumes have been extremely influential, summarizing the state of algebraic geometry at the time and pointing to future developments. The most recent Summer Institute in Algebraic Geometry was held July 2015 at the University of Utah in Salt Lake City, sponsored by the AMS with the collaboration of the Clay Mathematics Institute. This volume includes surveys growing out of plenary lectures and seminar talks during the meeting. Some present a broad overview of their topics, while others develop a distinctive perspective on an emerging

topic. Topics span both complex algebraic geometry and arithmetic questions, specifically, analytic techniques, enumerative geometry, moduli theory, derived categories, birational geometry, tropical geometry, Diophantine questions, geometric representation theory, characteristic and  $p$ -adic tools, etc. The resulting articles will be important references in these areas for years to come.

This edited collection of chapters, authored by leading experts, provides a complete and essentially self-contained construction of 3-fold and 4-fold klt flips. A large part of the text is a digest of Shokurov's work in the field and a concise, complete and pedagogical proof of the existence of 3-fold flips is presented. The text includes a ten page glossary and is accessible to students and researchers in algebraic geometry.

While Eugenio Calabi is best known for his contributions to the theory of Calabi-Yau manifolds, this Steele-Prize-winning geometer's fundamental contributions to mathematics have been far broader and more diverse than might be guessed from this one aspect of his work. His works have deep influence and lasting impact in global differential geometry, mathematical physics and beyond. By bringing together 47 of Calabi's important articles in a single volume, this book provides a comprehensive overview of his mathematical oeuvre, and includes papers on complex manifolds, algebraic geometry, Kähler metrics, affine geometry, partial differential equations, several complex variables, group actions and topology. The volume also includes essays on Calabi's mathematics by several of his mathematical admirers, including S.K. Donaldson, B. Lawson and S.-T. Yau, Marcel Berger; and Jean Pierre Bourguignon. This book is intended for mathematicians and graduate students around the world. Calabi's visionary contributions will certainly continue to shape the course of this subject far into the future.

Number Theory and Algebraic Geometry Cambridge University Press

Visionary articles explaining approaches to important problems on the interface of pure mathematics and mathematical physics.

Fascinating and surprising developments are taking place in the classification of algebraic varieties. The work of Hacon and McKernan and many others is causing a wave of breakthroughs in the minimal model program: we now know that for a smooth projective variety the canonical ring is finitely generated. These new results and methods are reshaping the field. Inspired by this exciting progress, the editors organized a meeting at Schiermonnikoog and invited leading experts to write papers about the recent developments. The result is the present volume, a lively testimony to the sudden advances that originate from these new ideas. This volume will be of interest to a wide range of pure mathematicians, but will appeal especially to algebraic and analytic geometers.

This book is a collection of selected papers written by researchers of our "RISC" institute (Research Institute for Symbolic Computation) along with the ESPRIT MEDLAR Project (Mechanizing Deduction in the Logics of Practical Reasoning). Naturally, the MEDLAR Project was and is the focal point for our institute whose main objective is the combination of foundational research in the area of symbolic computation and possible applications thereof for high-tech industrial projects. I am grateful to the director of the MEDLAR project, Jim Cunningham, for his enthusiasm, profound expertise, and continuous effort to manage a fruitful cooperation between various European working groups in the area of the project and for giving us the opportunity to be part of this challenging endeavor. I also acknowledge and feel indebted to Jochen Pfalzgraf for managing the RISC part of the MEDLAR project and to both him and Dongming Wang for editing this volume and organizing the refereeing process.

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