

## A Moscow Math Circle By Sergey Dorichenko

These two books are the first volumes of articles published from 1970 to 1990 in the Russian journal, *Kvant*. The influence of the this magazine on mathematics and physics education in Russia is unmatched. Articles selected for these two volumes are written by leading Russian mathematicians and expositors.

This book is devoted to billiards in their relation with differential geometry, classical mechanics, and geometrical optics. The book is based on an advanced undergraduate topics course (but contains more material than can be realistically taught in one semester). Although the minimum prerequisites include only the standard material usually covered in the first two years of college (the entire calculus sequence, linear algebra), readers should show some mathematical maturity and strongly rely on their mathematical common sense. As a reward, they will be taken to the forefront of current research.

What kind of book is this? It is a book produced by a remarkable cultural circumstance in the former Soviet Union which fostered the creation of groups of students, teachers, and mathematicians called "mathematical circles". The work is predicated on the idea that studying mathematics can generate the same enthusiasm as playing a team sport - without necessarily being competitive. This book is intended for both students and teachers who love mathematics and want to study its various branches beyond the limits of school curriculum.

This book is a translation from Russian of Part II of the book *Mathematics Through Problems: From Olympiads and Math Circles to Profession*. Part I, Algebra, was recently published in the same series. Part III, Combinatorics, will be published soon. The main goal of this book is to develop important parts of mathematics through problems. The authors tried to put together sequences of problems that allow high school students (and some undergraduates) with strong interest in mathematics to discover and recreate much of elementary mathematics and start edging into more sophisticated topics such as projective and affine geometry, solid geometry, and so on, thus building a bridge between standard high school exercises and more intricate notions in geometry. Definitions and/or references for material that is not standard in the school curriculum are included. To help students that might be unfamiliar with new material, problems are carefully arranged to provide gradual introduction into each subject. Problems are often accompanied by hints and/or complete solutions. The book is based on classes taught by the authors at different times at the Independent University of Moscow, at a number of Moscow schools and math circles, and at various summer schools. It can be used by high school students and undergraduates, their teachers, and organizers of summer camps and math circles. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.

The book teaches kindergarten and 1st grade students to break (positive integral) numbers into parts in all the possible ways. The book uses the developed technique to explain (prove) commutativity of addition of positive integers. The book also explores the concepts of digits and numbers, odd and even numbers, operations (functions), and inverse operations in an age-appropriate fashion. The book was tried and tested at Los Angeles Math Circle (LAMC), a free Sunday math school for mathematically inclined children run by UCLA Department of Mathematics. The book was used as a basis for a year-long enhancement math course at a variety of other locations, from math circles and after-school programs to a full-fledged elementary school. The second edition of the book is a feedback-based improvement of the first edition. It has two extra chapters, more than sixty extra problems, solutions to harder problems, ten quizzes, and more. The second edition has two parts. This book is Part 1.

This book is based on selected topics that the authors taught in math circles for elementary school students at the University of California, Berkeley; Stanford University; Dominican University (Marin County, CA); and the University of Oregon (Eugene). It is intended for people who are already running a math circle or who are thinking about organizing one. It can be used by parents to help their motivated, math-loving kids or by elementary school teachers. We also hope that bright fourth or fifth graders will be able to read this book on their own. The main features of this book are the logical sequence of the problems, the description of class reactions, and the hints given to kids when they get stuck. This book tries to keep the balance between two goals: inspire readers to invent their own original approaches while being detailed enough to work as a fallback in case the teacher needs to prepare a lesson on short notice. It introduces kids to combinatorics, Fibonacci numbers, Pascal's triangle, and the notion of area, among other things. The authors chose topics with deep mathematical context. These topics are just as engaging and entertaining to children as typical "recreational math" problems, but they can be developed deeper and to more advanced levels. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.

A Moscow Math Circle  
Week-by-week Problem Sets  
American Mathematical Soc.

"One of the themes of the book is how to have a fulfilling professional life. In order to achieve this goal, Krantz discusses keeping a vigorous scholarly program going and finding new challenges, as well as dealing with the everyday tasks of research, teaching, and administration." "In short, this is a survival manual for the professional mathematician - both in academics and in industry and government agencies. It is a sequel to the author's *A Mathematician's Survival Guide*."--BOOK JACKET.

The three most basic shapes -- squares, triangles and circles -- are all around us, from the natural world to the one we've engineered. Full of fascinating facts about these shapes and their 3D counterparts, *Shapes in Math, Science and Nature* introduces young readers to the basics of geometry and reveals its applications at home, school and everywhere in between. Puzzles and activities add to the fun factor.

Over 300 challenging problems in algebra, arithmetic, elementary number theory and trigonometry, selected from Mathematical Olympiads held at Moscow University. Only high school math needed. Includes complete solutions. Features 27 black-and-white illustrations. 1962 edition.

In recent decades it has become obvious that mathematics has always been a worldwide activity. But this is the first book to provide a substantial collection of English

translations of key mathematical texts from the five most important ancient and medieval non-Western mathematical cultures, and to put them into full historical and mathematical context. The Mathematics of Egypt, Mesopotamia, China, India, and Islam gives English readers a firsthand understanding and appreciation of these cultures' important contributions to world mathematics. The five section authors--Annette Imhausen (Egypt), Eleanor Robson (Mesopotamia), Joseph Dauben (China), Kim Plofker (India), and J. Lennart Berggren (Islam)--are experts in their fields. Each author has selected key texts and in many cases provided new translations. The authors have also written substantial section introductions that give an overview of each mathematical culture and explanatory notes that put each selection into context. This authoritative commentary allows readers to understand the sometimes unfamiliar mathematics of these civilizations and the purpose and significance of each text. Addressing a critical gap in the mathematics literature in English, this book is an essential resource for anyone with at least an undergraduate degree in mathematics who wants to learn about non-Western mathematical developments and how they helped shape and enrich world mathematics. The book is also an indispensable guide for mathematics teachers who want to use non-Western mathematical ideas in the classroom.

Challenge And Thrill Of Pre-College Mathematics Is An Unusual Enrichment Text For Mathematics Of Classes 9, 10, 11 And 12 For Use By Students And Teachers Who Are Not Content With The Average Level That Routine Text Dare Not Transcend In View Of Their Mass Clientele. It Covers Geometry, Algebra And Trigonometry Plus A Little Of Combinatorics. Number Theory And Probability. It Is Written Specifically For The Top Half Whose Ambition Is To Excel And Rise To The Peak Without Finding The Journey A Forced Uphill Task. The Undercurrent Of The Book Is To Motivate The Student To Enjoy The Pleasures Of A Mathematical Pursuit And Of Problem Solving. More Than 300 Worked Out Problems (Several Of Them From National And International Olympiads) Share With The Student The Strategy, The Excitement, Motivation, Modeling, Manipulation, Abstraction, Notation And Ingenuity That Together Make Mathematics. This Would Be The Starting Point For The Student, Of A Life-Long Friendship With A Sound Mathematical Way Of Thinking. There Are Two Reasons Why The Book Should Be In The Hands Of Every School Or College Student, (Whether He Belongs To A Mathematics Stream Or Not) One, If He Likes Mathematics And, Two, If He Does Not Like Mathematics- The Former, So That The Cramped Robot-Type Treatment In The Classroom Does Not Make Him Into The Latter; And The Latter So That By The Time He Is Halfway Through The Book, He Will Invite Himself Into The Former.

This volume completes the English adaptation of a classical Russian textbook in elementary Euclidean geometry. The 1st volume subtitled "Book I. Planimetry" was published in 2006 (ISBN 0977985202). This 2nd volume (Book II. Stereometry) covers solid geometry, and contains a chapter on vectors, foundations, and introduction in non-Euclidean geometry added by the translator. The book intended for high-school and college students, and their teachers. Includes 317 exercises, index, and bibliography.

This book is a translation from Russian of Part I of the book Mathematics Through Problems: From Olympiads and Math Circles to Profession. The other two parts, Geometry and Combinatorics, will be published soon. The main goal of this book is to develop important parts of mathematics through problems. The author tries to put together sequences of problems that allow high school students (and some undergraduates) with strong interest in mathematics to discover and recreate much of elementary mathematics and start edging into the sophisticated world of topics such as group theory, Galois theory, and so on, thus building a bridge (by showing that there is no gap) between standard high school exercises and more intricate and abstract concepts in mathematics. Definitions and/or references for material that is not standard in the school curriculum are included. However, many topics in the book are difficult when you start learning them from scratch. To help with this, problems are carefully arranged to provide gradual introduction into each subject. Problems are often accompanied by hints and/or complete solutions. The book is based on classes taught by the author at different times at the Independent University of Moscow, at a number of Moscow schools and math circles, and at various summer schools. It can be used by high school students and undergraduates, their teachers, and organizers of summer camps and math circles. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.

Vladimir Arnold (1937-2010) was one of the great mathematical minds of the late 20th century. He did significant work in many areas of the field. On another level, he was keeping with a strong tradition in Russian mathematics to write for and to directly teach younger students interested in mathematics. This book contains some examples of Arnold's contributions to the genre. "Continued Fractions" takes a common enrichment topic in high school math and pulls it in directions that only a master of mathematics could envision. "Euler Groups" treats a similar enrichment topic, but it is rarely treated with the depth and imagination lavished on it in Arnold's text. He sets it in a mathematical context, bringing to bear numerous tools of the trade and expanding the topic way beyond its usual treatment. In "Complex Numbers" the context is physics, yet Arnold artfully extracts the mathematical aspects of the discussion in a way that students can understand long before they master the field of quantum mechanics. "Problems for Children 5 to 15 Years Old" must be read as a collection of the author's favorite intellectual morsels. Many are not original, but all are worth thinking about, and each requires the solver to think out of his or her box. Dmitry Fuchs, a long-term friend and collaborator of Arnold, provided solutions to some of the problems. Readers are of course invited to select their own favorites and construct their own favorite solutions. In reading these essays, one has the sensation of walking along a path that is found to ascend a mountain peak and then being shown a vista whose existence one could never suspect from the ground. Arnold's style of exposition is unforgiving. The reader--even a professional mathematician--will find paragraphs that require hours of thought to unscramble, and he or she must have patience with the ellipses of thought and the leaps of reason. These are all part of Arnold's intent. In the

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Number Theory has fascinated mathematicians from the most ancient of times. A remarkable feature of number theory is the fact that there is something in it for everyone from puzzle enthusiasts, problem solvers and amateur mathematicians to professional scientists and technologists.

Math circles provide a setting in which mathematicians work with secondary school students who are interested in mathematics. This form of outreach, which has existed for decades in Russia, Bulgaria, and other countries, is now rapidly spreading across the United States as well. The first part of this book offers helpful advice on all aspects of math circle operations, culled from conversations with over a dozen directors of successful math circles. Topics include creative means for getting the word out to students, sound principles for selecting effective speakers, guidelines for securing financial support, and tips for designing an exciting math circle session. The purpose of this discussion is to enable math circle coordinators to establish a thriving group in which students can experience the delight of mathematical investigation. The second part of the book outlines ten independent math circle sessions, covering a variety of topics and difficulty levels. Each chapter contains detailed presentation notes along with a useful collection of problems and solutions. This book will be an indispensable resource for any individual involved with a math circle or anyone who would like to see one begin in his or her community. Sam Vandervelde teaches at St. Lawrence University. He launched the Stanford Math Circle and also writes and coordinates the Mandelbrot Competition, a math contest for high schools. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession. Titles in this series are co-published with the Mathematical Sciences Research Institute (MSRI).

The people of the Navajo Nation know mathematics education for their children is essential. They were joined by mathematicians familiar with ways to deliver problems and a pedagogy that, through exploration, shows the art, joy and beauty in mathematics. This combined effort produced a series of Navajo Math Circles—interactive mathematical explorations—across the Navajo Reservation. This book contains the mathematical details of that effort. Between its covers is a thematic rainbow of problem sets that were used in Math Circle sessions on the Reservation. The problem sets are good for puzzling over and exploring the mathematical ideas within. They will help nurture curiosity and confidence in students. The problems come with suggestions for pacing, for adjusting the problems to be more or less challenging, and for different approaches to solving them. This book is a wonderful resource for any teacher wanting to enrich the mathematical lives of students and for anyone curious about mathematical thinking outside the box. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.

This volume of research papers is an outgrowth of the Manin Seminar at Moscow University, devoted to K-theory, homological algebra and algebraic geometry. The main topics discussed include additive K-theory, cyclic cohomology, mixed Hodge structures, theory of Virasoro and Neveu-Schwarz algebras.

Early middle school is a great time for children to start their mathematical circle education. This time is a period of curiosity and openness to learning. The thinking habits and study skills acquired by children at this age stay with them for a lifetime. Mathematical circles, with their question-driven approach and emphasis on creative problem-solving, have been rapidly gaining popularity in the United States. The circles expose children to the type of mathematics that stimulates development of logical thinking, creativity, analytical abilities and mathematical reasoning. These skills, while scarcely touched upon at school, are in high demand in the modern world. This book contains everything that is needed to run a successful mathematical circle for a full year. The materials, distributed among 29 weekly lessons, include detailed lectures and discussions, sets of problems with solutions, and contests and games. In addition, the book shares some of the know-how of running a mathematical circle. The curriculum, which is based on the rich and long-standing Russian math circle tradition, has been modified and adapted for teaching in the United States. For the past decade, the author has been actively involved in teaching a number of mathematical circles in the Seattle area. This book is based on her experience and on the compilation of materials from these circles. The material is intended for students in grades 5 to 7. It can be used by teachers and parents with various levels of expertise who are interested in teaching mathematics with the emphasis on critical thinking. Also, this book will be of interest to mathematically motivated children. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.

Many mathematicians have been drawn to mathematics through their experience with math circles. The Berkeley Math Circle (BMC) started in 1998 as one of the very first math circles in the U.S. Over the last decade and a half, 100 instructors--university professors, business tycoons, high school teachers, and more--have shared their passion for mathematics by delivering over 800 BMC sessions on the UC Berkeley campus every week during the school year. This second volume of the book series is based on a dozen of these sessions, encompassing a variety of enticing and stimulating mathematical topics, some new and some continuing from Volume I: from dismantling Rubik's Cube and randomly putting it back together to solving it with the power of group theory; from raising knot-eating machines and letting Alexander the Great cut the Gordian Knot to breaking through knot theory via the Jones polynomial; from entering a seemingly hopeless infinite raffle to becoming friendly with multiplicative functions in the land of Dirichlet, Möbius, and Euler; from leading an army of jumping fleas in an old problem from the International Mathematical Olympiads to improving our own essay-writing strategies; from searching for optimal paths on a hot summer day to questioning whether Archimedes was on his way to discovering trigonometry 2000 years ago Do some of these scenarios sound bizarre, having never before been associated with mathematics? Mathematicians love having fun while doing serious mathematics and that love is what this book intends to share with the reader. Whether at a beginner, an intermediate, or an advanced level, anyone can find a place here to be provoked to think deeply and to be inspired to create. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession. Titles in this series are co-published with the Mathematical Sciences Research Institute (MSRI).

This book is a captivating account of a professional mathematician's experiences conducting a math circle for preschoolers in his apartment in Moscow in the 1980s. As anyone who has taught or raised young children knows, mathematical education for little kids is a real mystery. What are they capable of? What should they learn first? How hard should they work? Should they even "work" at all? Should we push them, or just let them be? There are no correct answers to these questions, and the author deals with them in classic math-circle style: he doesn't ask and then answer a question, but shows us a problem--be it mathematical or pedagogical--and describes to us what happened. His book is a narrative about what he did, what he tried, what worked, what failed, but most important, what the kids experienced. This book does not purport to show you how to create precocious high achievers. It is just one person's story about things he tried with a half-dozen young children. Mathematicians, psychologists, educators, parents, and everybody interested in the intellectual development in young children will find this book to be an invaluable, inspiring resource. In the interest of fostering a greater

awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession. Titles in this series are co-published with the Mathematical Sciences Research Institute (MSRI).

Based on Stanford University's well-known competitive exam, this excellent mathematics workbook offers students at both high school and college levels a complete set of problems, hints, and solutions. 1974 edition.

Math Hour Olympiads is a non-standard method of training middle- and high-school students interested in mathematics where students spend several hours thinking about a few difficult and unusual problems. When a student solves a problem, the solution is presented orally to a pair of friendly judges. Discussing the solutions with the judges creates a personal and engaging mathematical experience for the students and introduces them to the true nature of mathematical proof and problem solving. This book recounts the authors' experiences from the first ten years of running a Math Hour Olympiad at the University of Washington in Seattle. The major part of the book is devoted to problem sets and detailed solutions, complemented by a practical guide for anyone who would like to organize an oral olympiad for students in their community. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.

An awesome, globe-spanning, and New York Times best-selling journey through the beauty and power of mathematics What if you had to take an art class in which you were only taught how to paint a fence? What if you were never shown the paintings of van Gogh and Picasso, weren't even told they existed? Alas, this is how math is taught, and so for most of us it becomes the intellectual equivalent of watching paint dry. In *Love and Math*, renowned mathematician Edward Frenkel reveals a side of math we've never seen, suffused with all the beauty and elegance of a work of art. In this heartfelt and passionate book, Frenkel shows that mathematics, far from occupying a specialist niche, goes to the heart of all matter, uniting us across cultures, time, and space. *Love and Math* tells two intertwined stories: of the wonders of mathematics and of one young man's journey learning and living it. Having braved a discriminatory educational system to become one of the twenty-first century's leading mathematicians, Frenkel now works on one of the biggest ideas to come out of math in the last 50 years: the Langlands Program. Considered by many to be a Grand Unified Theory of mathematics, the Langlands Program enables researchers to translate findings from one field to another so that they can solve problems, such as Fermat's last theorem, that had seemed intractable before. At its core, *Love and Math* is a story about accessing a new way of thinking, which can enrich our lives and empower us to better understand the world and our place in it. It is an invitation to discover the magic hidden universe of mathematics.

The Moscow Mathematical Olympiad has been challenging high school students with stimulating, original problems of different degrees of difficulty for over 75 years. The problems are nonstandard; solving them takes wit, thinking outside the box, and, sometimes, hours of contemplation. Some are within the reach of most mathematically competent high school students, while others are difficult even for a mathematics professor. Many mathematically inclined students have found that tackling these problems, or even just reading their solutions, is a great way to develop mathematical insight. In 2006 the Moscow Center for Continuous Mathematical Education began publishing a collection of problems from the Moscow Mathematical Olympiads, providing for each an answer (and sometimes a hint) as well as one or more detailed solutions. This volume represents the years 1993-1999. The problems and the accompanying material are well suited for math circles. They are also appropriate for problem-solving classes and practice for regional and national mathematics competitions. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession. Titles in this series are co-published with the Mathematical Sciences Research Institute (MSRI).

This is, quite simply, the best and most popular puzzle book ever published in the Soviet Union. Since its first appearance in 1956 there have been eight editions as well as translations from the original Russian into Ukrainian, Estonian, Lettish, and Lithuanian. Almost a million copies of the Russian version alone have been sold. Part of the reason for the book's success is its marvelously varied assortment of brainteasers ranging from simple "catch" riddles to difficult problems (none, however, requiring advanced mathematics). Many of the puzzles will be new to Western readers, while some familiar problems have been clothed in new forms. Often the puzzles are presented in the form of charming stories that provide non-Russian readers with valuable insights into contemporary Russian life and customs. In addition, Martin Gardner, former editor of the *Mathematical Games* Department, *Scientific American*, has clarified and simplified the book to make it as easy as possible for an English-reading public to understand and enjoy. He has been careful, moreover, to retain nearly all the freshness, warmth, and humor of the original. Lavishly illustrated with over 400 clear diagrams and amusing sketches, this inexpensive edition of the first English translation will offer weeks or even months of stimulating entertainment. It belongs in the library of every puzzlist or lover of recreational mathematics.

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Classical Euclidean geometry, with all its triangles, circles, and inscribed angles, remains an excellent playground for high-school mathematics students, even if it looks outdated from the professional mathematician's viewpoint. It provides an excellent choice of elegant and natural problems that can be used in a course based on problem solving. The book contains more than 750 (mostly) easy but nontrivial problems in all areas of plane geometry and solutions for most of them, as well as additional problems for self-study (some with hints). Each chapter also provides concise reminders of basic notions used in the chapter, so the book is almost self-contained (although a good textbook and competent teacher are always recommended). More than 450 figures illustrate the problems and their solutions. The book can be used by motivated high-school students, as well as their teachers and parents. After solving the problems in the book the student will have mastered the main notions and methods of plane geometry and, hopefully, will have had fun in the process. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession. What a joy! Shen's "Geometry in Problems" is a gift to the school teaching world. Beautifully organized by content topic, Shen has collated a vast collection of fresh, innovative, and highly classroom-relevant questions, problems, and challenges sure to enliven the minds and clever thinking of all those studying Euclidean geometry for the first time. This book is a spectacular resource for educators and students alike. Users will not only sharpen their mathematical understanding of specific topics but will also sharpen their problem-solving wits and come to truly own the mathematics explored. Also, Math Circle leaders can draw much inspiration for session ideas from the material presented in this book. --James Tanton, Mathematician-at-Large, Mathematical Association of America We learn mathematics best by doing mathematics. The author of this book recognizes this principle. He invites the reader to participate in learning plane geometry through carefully chosen problems, with brief explanations leading to much activity. The problems in the book are sometimes deep and subtle: almost everyone can do some of them, and almost no one can do all. The reader comes away with a view of geometry refreshed by experience.

--Mark Saul, Director of Competitions, Mathematical Association of America

Delve into the development of modern mathematics and match wits with Euclid, Newton, Descartes, and others. Each chapter explores an individual type of challenge, with commentary and practice problems. Solutions.

The main part of this book describes the first semester of the existence of a successful and now highly popular program for elementary school students at the Berkeley Math Circle. The topics discussed in the book introduce the participants to the basics of many important areas of modern mathematics, including logic, symmetry, probability theory, knot theory, cryptography, fractals, and number theory. Each chapter in the first part of this book consists of two parts. It starts with generously illustrated sets of problems and hands-on activities. This part is addressed to young readers who can try to solve problems on their own or to discuss them with adults. The second part of each chapter is addressed to teachers and parents. It includes comments on the topics of the lesson, relates those topics to discussions in other chapters, and describes the actual reaction of math circle participants to the proposed activities. The supplementary problems that were discussed at workshops of Math Circle at Kansas State University are given in the second part of the book. The book is richly illustrated, which makes it attractive to its young audience. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession. Titles in this series are co-published with the Mathematical Sciences Research Institute (MSRI).

Geometry has been an essential element in the study of mathematics since antiquity. Traditionally, we have also learned formal reasoning by studying Euclidean geometry. In this book, David Clark develops a modern axiomatic approach to this ancient subject, both in content and presentation. Mathematically, Clark has chosen a new set of axioms that draw on a modern understanding of set theory and logic, the real number continuum and measure theory, none of which were available in Euclid's time. The result is a development of the standard content of Euclidean geometry with the mathematical precision of Hilbert's foundations of geometry. In particular, the book covers all the topics listed in the Common Core State Standards for high school synthetic geometry. The presentation uses a guided inquiry, active learning pedagogy. Students benefit from the axiomatic development because they themselves solve the problems and prove the theorems with the instructor serving as a guide and mentor. Students are thereby empowered with the knowledge that they can solve problems on their own without reference to authority. This book, written for an undergraduate axiomatic geometry course, is particularly well suited for future secondary school teachers. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.

The ARML (American Regions Math League) Power Contest is truly a unique competition in which a team of students is judged on its ability to discover a pattern, express the pattern in precise mathematical language, and provide a logical proof of its conjectures. Just as a team of students can be self-directed to solve each problem set, a teacher, math team coach, or math circle leader could take these ideas and questions and lead students into problem solving and mathematical discovery. This book contains thirty-seven interesting and engaging problem sets from the ARML Power Contests from 1994 to 2013. They are generally extensions of the high school mathematics classroom and often connect two remote areas of mathematics. Additionally, they provide meaningful problem situations for both the novice and

the veteran mathlete. Thomas Kilkelly has been a mathematics teacher for forty-three years. During that time he has been awarded several teaching honors and has coached many math teams to state and national championships. He has always been an advocate for more discovery, integration, and problem solving in the mathematics classroom. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession. Titles in this series are co-published with the Mathematical Sciences Research Institute (MSRI).

Functions and their properties have been part of the rigorous precollege curriculum for decades. And functional equations have been a favorite topic of the leading national and international mathematical competitions. Yet the subject has not received equal attention by authors at an introductory level. The majority of the books on the topic remain unreachable to the curious and intelligent precollege student. The present book is an attempt to eliminate this disparity. The book opens with a review chapter on functions, which collects the relevant foundational information on functions, plus some material potentially new to the reader. The next chapter presents a working definition of functional equations and explains the difficulties in trying to systematize the theory. With each new chapter, the author presents methods for the solution of a particular group of equations. Each chapter is complemented with many solved examples, the majority of which are taken from mathematical competitions and professional journals. The book ends with a chapter of unsolved problems and some other auxiliary material. The book is an invaluable resource for precollege and college students who want to deepen their knowledge of functions and their properties, for teachers and instructors who wish to enrich their curricula, and for any lover of mathematical problem-solving techniques. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.

Held annually in Moscow since 1990, the Mathematical Festival is a brilliant and fascinating math competition attended by hundreds of middle school students. Participants of the Festival solve interesting mathematical problems and partake in other engaging activities, while cultivating key skills such as intuitive reasoning and quick thinking. This book contains problems presented at the Festival during the years 1990-2011, along with hints and solutions for many of them. Most of the problems are accessible to students with no additional training in mathematics and may be used as supplementary material at school or at home. Other problems, however, are more advanced and will be enjoyed by students with a deeper interest in mathematics. Most of the problems in this book are specially created for Mathematical Festival competitions by leading Russian experts in school and extracurricular math education and have never been published before. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.

Moscow has a rich tradition of successful math circles, to the extent that many other circles are modeled on them. This book presents materials used during the course of one year in a math circle organized by mathematics faculty at Moscow State University, and also used at the mathematics magnet school known as Moscow School Number 57. Each problem set has a similar structure: it combines review material with a new topic, offering problems in a range of difficulty levels. This time-tested pattern has proved its effectiveness in engaging all students and helping them master new material while building on earlier knowledge. The introduction describes in detail how the math circles at Moscow State University are run. Dorichenko describes how the early sessions differ from later sessions, how to choose problems, and what sorts of difficulties may arise when running a circle. The book also includes a selection of problems used in the competition known as the Mathematical Maze, a mathematical story based on actual lessons with students, and an addendum on the San Jose Mathematical Circle, which is run in the Russian style. In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.

Mathematical Olympiad Challenges is a rich collection of problems put together by two experienced and well-known professors and coaches of the U.S. International Mathematical Olympiad Team. Hundreds of beautiful, challenging, and instructive problems from algebra, geometry, trigonometry, combinatorics, and number theory were selected from numerous mathematical competitions and journals. An important feature of the work is the comprehensive background material provided with each grouping of problems. The problems are clustered by topic into self-contained sections with solutions provided separately. All sections start with an essay discussing basic facts and one or two representative examples. A list of carefully chosen problems follows and the reader is invited to take them on. Additionally, historical insights and asides are presented to stimulate further inquiry. The emphasis throughout is on encouraging readers to move away from routine exercises and memorized algorithms toward creative solutions to open-ended problems. Aimed at motivated high school and beginning college students and instructors, this work can be used as a text for advanced problem-solving courses, for self-study, or as a resource for teachers and students training for mathematical competitions and for teacher professional development, seminars, and workshops.

The papers collected in this volume reflect some of the directions of research in two closely related fields: Complex Dynamics and Renormalization in Dynamical Systems. While dynamics of polynomial mappings, particularly quadratics, has by now reached a mature state of development, much less is known about non-polynomial rational maps. The reader will be introduced into this fascinating world and a related area of transcendental dynamics by the papers in this volume. A graduate student will find an area rich with open problems and beautiful computer simulations. A survey by V. Nekrashevych introduces the reader to iterated monodromy groups of rational mappings, a recently developed subject that links geometric group theory to combinatorics of rational maps. In this new language, many questions related to Thurston's theory of branched coverings of the sphere can be answered explicitly. Renormalization theory occupies a central place in modern Complex Dynamics. The progress in understanding the structure of the Mandelbrot set, polynomial Julia sets, and Feigenbaum-type universalities stems from renormalization techniques. Renormalization of circle maps and rotation domains, such as Siegel disks, can be understood in the context of the classical KAM theory. Corresponding phenomena in higher dimensions, such as universal scaling in area-preserving maps in 2D, on the boundary of KAM, pose a challenging problem. A survey by H. Koch and several other papers in the volume will introduce the reader to this direction of study.

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