A Mathematical Introduction To Logic Second Edition

This lucid, non-intimidating presentation by a Russian scholar explores propositional logic, propositional calculus, and predicate logic. Topics include computer science and systems analysis, linguistics, and problems in the foundations of mathematics. Accessible to high school students, it also constitutes a valuable review of fundamentals for professionals. 1970 edition.

This book is intended for mathematicians. Its origins lie in a course of lectures given by an algebraist to a class which had just completed a sub stantial course on abstract algebra. Consequently, our treatment ofthe sub ject is algebraic. Although we assurne a reasonable level of sophistication in algebra, the text requires little more than the basic notions of group, ring, module, etc. A more detailed knowledge of algebra is required for some of . the exercises. We also assurne a familiarity with the main ideas of set theory, including cardinal numbers and Zorn's Lemma. In this book, we carry out a mathematical study of the logic used in mathematics. We do this by constructing a mathematical model oflogic and applying mathematics to analyse the properties of the model. We therefore regard all our existing knowledge of mathematics as being applicable to the analysis of the model, and in particular we accept set theory as part of the meta-Ianguage. We are not attempting to construct a foundation on which all mathematics is to be based-rather, any conclusions to be drawn about the foundations of mathematics co me only by analogy with the model, and are to be regarded in much the same way as the conclusions drawn from any scientific theory.

1. This book is above all addressed to mathematicians. It is intended to be a textbook of mathematical logic on a sophisticated level, presenting the reader with several of the most significant discoveries of the last ten or fifteen years. These include: the independence of the continuum hypothe sis, the Diophantine nature of enumerable sets, the impossibility of finding an algorithmic solution for one or two old problems. All the necessary preliminary material, including predicate logic and the fundamentals of recursive function theory, is presented systematically and with complete proofs. We only assume that the reader is familiar with "naive" set theoretic arguments. In this book mathematical logic is presented both as a part of mathe matics and as the result of its self-perception. Thus, the substance of the book consists of difficult proofs of subtle theorems, and the spirit of the book consists of attempts to explain what these theorems say about the mathematical way of thought. Foundational problems are for the most part passed over in silence. Most likely, logic is capable of justifying mathematics to no greater extent than biology is capable of justifying life. 2. The first two chapters are devoted to predicate logic. The presenta tion here is fairly standard, except that semantics occupies a very domi nant position, truth is introduced before deducibility, and models of speech in formal languages precede the systematic study of syntax.

This is a compact mtroduction to some of the pnncipal tOpICS of mathematical logic . In the belief that beginners should be exposed to the most natural and easiest proofs, I have used free-swinging set-theoretic methods. The significance of a demand for constructive proofs can be evaluated only after a certain amount of experience with mathematical logic has been obtained. If we are to be expelled from "Cantor's paradise" (as nonconstructive set theory was called by Hilbert), at least we should know what we are missing. The major changes in this new edition are the following. (1) In Chapter 5, Effective Computability, Turing-computablIity IS now the central notion, and diagrams (flow-charts) are used to construct Turing machines. There are also treatments of Markov algorithms, Herbrand-Godel-computability, register machines, and random access machines. Recursion theory is gone into a little more deeply, including the s-m-n theorem, the recursion theorem, and Rice's Theorem. (2) The proofs of the Incompleteness Theorems are now based upon the Diagonalization Lemma. Lob's Theorem and its connection with Godel's Second Theorem are also studied. (3) In Chapter 2, Quantification Theory, Henkin's proof of the completeness theorem has been postponed until the reader has gained more experience in proof techniques. The exposition of the proof itself has been improved by breaking it down into smaller pieces and using the notion of a scapegoat theory. There is also an entirely new section on semantic trees.

Written by a pioneer of mathematical logic, this comprehensive graduate-level text explores the constructive theory of first-order predicate calculus. It covers formal methods — including algorithms and epitheory — and offers a brief treatment of Markov's approach to algorithms. It also explains elementary facts about lattices and similar algebraic systems. 1963 edition.

Mathematical Logic is a collection of the works of one of the leading figures in 20th-century science. This collection of A.M. Turing's works is intended to include all his mature scientific writing, including a substantial quantity of unpublished material. His work in pure mathematics and mathematical logic extended considerably further; the work of his last years, on morphogenesis in plants, is also of the greatest originality and of permanent importance. This book is divided into three parts. The first part focuses on computability and ordinal logics and covers Turing's work between 1937 and 1938. The second part covers type theory; it provides a general introduction to Turing's work on type theory and covers his published and unpublished works between 1941 and 1948. Finally, the third part focuses on enigmas, mysteries, and loose ends. This concluding section of the book discusses Turing's Treatise on the Enigma, with excerpts from the Enigma Paper. It also delves into Turing's papers on programming and on minimum cost sequential analysis, featuring an excerpt from the unpublished manuscript. This book will be of interest to mathematicians, logicians, and computer scientists.

This introduction to first-order logic clearly works out the role of first-order logic in the foundations of mathematics, particularly the two basic questions of the range of the axiomatic method and of theorem-proving by machines. It covers several advanced topics not commonly treated in introductory texts, such as Fraïssé's characterization of elementary equivalence, Lindström's theorem on the maximality of first-order logic, and the fundamentals of logic programming.

Assuming no previous study in logic, this informal yet rigorous text covers the material of a standard undergraduate first course in mathematical logic, using natural deduction and leading up to the completeness theorem for first-order logic. At each stage of the text, the reader is given an intuition based on standard mathematical practice, which is subsequently developed with clean formal mathematics. Alongside the practical examples, readers learn what can and can't becalculated; for example the correctness of a derivation proving a given sequent can be tested mechanically, but there is no general mechanical test for the existence of a derivation proving the given sequent. The undecidability results are proved rigorously in an optional final chapter, assumingMatiyasevich's theorem characterising the computably enumerable relations. Rigorous proofs of the adequacy and completeness proofs of the relevant logics are provided, with careful attention to the languages involved. Optional sections discuss the classification of mathematical structures by first-order theories; the required theory of cardinality is developed from scratch. Throughout the book there are notes on historical aspects of the material, and connections with linguistics andcomputer science, and the discussion of syntax and semantics is influenced by modern linguistic approaches. Two basic themes in recent cognitive science studies of actual human reasoning are also introduced. Including extensive exercises and selected solutions, this text is ideal for students in Logic,Mathematics, Philosophy, and Computer Science.

This book gives a mathematical treatment of the basic ideas and results of logic. It is intended to serve as a textbook for an introductory mathematics course in logic at the junior-senior level. The objectives are to present the important concepts and theorems of logic and to explain their significance and their relationship to the reader's other mathematical work.

This classic introduction to the main areas of mathematical logic provides the basis for a first graduate course in the subject. It embodies the viewpoint that mathematical logic is not a collection of vaguely related results, but a coherent method of attacking some of the most interesting problems, which face the mathematician. The author presents the basic concepts in an unusually clear and accessible fashion, concentrating on what he views as the central topics of mathematical logic: proof theory, model theory, recursion theory, axiomatic number theory, and set theory. There are many exercises, and they provide the outline of what amounts to a second book that goes into all topics in more depth. This book has played a role in the education of many mature and accomplished researchers.

Contents include an elementary but thorough overview of mathematical logic of 1st order; formal number theory; surveys of the work by Church, Turing, and others, including Gödel's completeness theorem, Gentzen's theorem, more.

A Mathematical Introduction to Logic, Second Edition, offers increased flexibility with topic coverage, allowing for choice in how to utilize the textbook in a course. The author has made this edition more accessible to better meet the needs of today's undergraduate mathematics and philosophy students. It is intended for the reader who has not studied logic previously, but who has some experience in mathematical reasoning. Material is presented on computer science issues such as computational complexity and database queries, with additional coverage of introductory material such as sets. * Increased flexibility of the text, allowing instructors more choice in how they use the textbook in courses. * Reduced mathematical rigour to fit the needs of undergraduate students

At the intersection of mathematics, computer science, and philosophy, mathematical logic examines the power and limitations of formal mathematical thinking. In this expansion of Leary's user-friendly 1st edition, readers with no previous study in the field are introduced to the basics of model theory, proof theory, and computability theory. The text is designed to be used either in an upper division undergraduate classroom, or for self study. Updating the 1st Edition's treatment of languages, structures, and deductions, leading to rigorous proofs of Godel's First and Second Incompleteness Theorems, the expanded 2nd Edition includes a new introduction to incompleteness through computability as well as solutions to selected exercises.

This book was written to serve as an introduction to logic, with in each chapter – if applicable – special emphasis on the interplay between logic and philosophy, mathematics, language and (theoretical) computer science. The reader will not only be provided with an introduction to classical logic, but to philosophical (modal, epistemic, deontic, temporal) and intuitionistic logic as well. The first chapter is an easy to read non-technical Introduction to the topics in the book. The next chapters are consecutively about Propositional Logic, Sets (finite and infinite), Predicate Logic, Arithmetic and Gödel’s Incompleteness Theorems, Modal Logic, Philosophy of Language, Intuitionism and Intuitionistic Logic, Applications (Prolog; Relational Databases and SQL; Social Choice Theory, in particular Majority Judgment) and finally, Fallacies and Unfair Discussion Methods. Throughout the text, the author provides some impressions of the historical development of logic: Stoic and Aristotelian logic, logic in the Middle Ages and Frege's Begriffsschrift, together with the works of George Boole (1815-1864) and August De Morgan (1806-1871), the origin of modern logic. Since "if ..., then ..." can be considered to be the heart of logic, throughout this book much attention is paid to conditionals: material, strict and relevant implication, entailment, counterfactuals and conversational implicature are treated and many references for further reading are given. Each chapter is concluded with answers to the exercises.

Mathematical logic developed into a broad discipline with many applications in mathematics, informatics, linguistics and philosophy. This text introduces the fundamentals of this field, and this new edition has been thoroughly expanded and revised.

Rigorous introduction is simple enough in presentation and context for wide range of students. Symbolizing sentences; logical inference; truth and validity; truth tables; terms, predicates, universal quantifiers; universal specification and laws of identity; more.

This book grew out of lectures. It is intended as an introduction to classical two-valued predicate logic. The restriction to classical logic is not meant to imply that this logic is intrinsically better than other, non-classical logics; however, classical logic is a good introduction to logic because of its simplicity, and a good basis for applications because it is the foundation of classical mathematics, and thus of the exact sciences which are based on it. The book is meant primarily for mathematics students who are already acquainted with some of the fundamental concepts of mathematics, such as that of a group. It should help the reader to see for himself the advantages of a formalisation. The step from the everyday language to a formalised language, which usually creates difficulties, is dis cussed and practised thoroughly. The analysis of the way in which basic mathematical structures are approached in mathematics leads in a natural way to the semantic notion of consequence. One of the substantial achievements of modern logic has been to show that the notion of consequence can be replaced by a provably equivalent notion of derivability which is defined by means of a calculus. Today we know of many calculi which have this property.

Demonstrating the different roles that logic plays in the disciplines of computer science, mathematics, and philosophy, this concise undergraduate textbook covers select topics from three different areas of logic: proof theory, computability theory, and nonclassical logic. The book balances accessibility, breadth, and rigor, and is designed so that its materials will fit into a single semester. Its distinctive presentation of traditional logic material will enhance readers' capabilities and mathematical maturity. The proof theory portion presents classical propositional logic and first-order logic using a computer-oriented (resolution) formal system. Linear resolution and its connection to the programming language Prolog are also treated. The computability component offers a machine model and mathematical model for computation, proves the equivalence of the two approaches, and includes famous decision problems unsolvable by an algorithm. The section on nonclassical logic discusses the shortcomings of classical logic in its treatment of implication and an alternate approach that improves upon it: Anderson and Belnap's relevance logic. Applications are included in each section. The material on a four-valued semantics for relevance logic is presented in textbook form for the first time. Aimed at upper-level undergraduates of moderate analytical background, Three Views of Logic will be useful in a variety of classroom settings. Gives an exceptionally broad view of logic Treats traditional logic in a modern format Presents relevance logic with applications Provides an ideal text for a variety of one-semester upper-level undergraduate courses

This book is intended as an undergraduate senior level or beginning graduate level text for mathematical logic. There are virtually no prere quisites, although a familiarity with notions encountered in a beginning course in abstract algebra such as groups, rings, and fields will be useful in providing some motivation for the topics in Part III. An attempt has been made to develop the beginning of each part slowly and then to gradually quicken the pace and the complexity of the material. Each part ends with a brief introduction to selected topics of current

interest. The text is divided into three parts: one dealing with set theory, another with computable function theory, and the last with model theory. Part III relies heavily on the notation, concepts and results discussed in Part I and to some extent on Part II. Parts I and II are independent of each other, and each provides enough material for a one semester course. The exercises cover a wide range of difficulty with an emphasis on more routine problems in the earlier sections of each part in order to familiarize the reader with the new notions and methods. The more difficult exercises are accompanied by hints. In some cases significant theorems are devel oped step by step with hints in the problems. Such theorems are not used later in the sequence.
Combining stories of great writers and philosophers with quotations and riddles, this completely original text for first courses in mathematical logic examines problems related to proofs, propositional logic and first-order logic, undecidability, and other topics. 2013 edition.

This comprehensive overview ofmathematical logic is designedprimarily for advanced undergraduatesand graduate studentsof mathematics. The treatmentalso contains much of interest toadvanced students in computerscience and philosophy. Topics include propositional logic;first-order languages and logic; incompleteness, undecidability,and indefinability; recursive functions; computability;and Hilbert’s Tenth Problem.Reprint of the PWS Publishing Company, Boston, 1995edition.

New corrected printing of a well-established text on logic at the introductory level.

In case you are considering to adopt this book for courses with over 50 students, please contact ties.nijssen@springer.com for more information. This introduction to mathematical logic starts with propositional calculus and first-order logic. Topics covered include syntax, semantics, soundness, completeness, independence, normal forms, vertical paths through negation normal formulas, compactness, Smullyan's Unifying Principle, natural deduction, cut-elimination, semantic tableaux, Skolemization, Herbrand's Theorem, unification, duality, interpolation, and definability. The last three chapters of the book provide an introduction to type theory (higher-order logic). It is shown how various mathematical concepts can be formalized in this very expressive formal language. This expressive notation facilitates proofs of the classical incompleteness and undecidability theorems which are very elegant and easy to understand. The discussion of semantics makes clear the important distinction between standard and nonstandard models which is so important in understanding puzzling phenomena such as the incompleteness theorems and Skolem's Paradox about countable models of set theory. Some of the numerous exercises require giving formal proofs. A computer program called ETPS which is available from the web facilitates doing and checking such exercises. Audience: This volume will be of interest to mathematicians, computer scientists, and philosophers in universities, as well as to computer scientists in industry who wish to use higher-order logic for hardware and software specification and verification.

A mathematical introduction to the theory and applications of logic and set theory with an emphasis on writing proofs Highlighting the applications and notations of basic mathematical concepts within the framework of logic and set theory, A First Course in Mathematical Logic and Set Theory introduces how logic is used to prepare and structure proofs and solve more complex problems. The book begins with propositional logic, including two-column proofs and truth table applications, followed by first-order logic, which provides the structure for writing mathematical proofs. Set theory is then introduced and serves as the basis for defining relations, functions, numbers, mathematical induction, ordinals, and cardinals. The book concludes with a primer on basic model theory with applications to abstract algebra. A First Course in Mathematical Logic and Set Theory also includes: Section exercises designed to show the interactions between topics and reinforce the presented ideas and concepts Numerous examples that illustrate theorems and employ basic concepts such as Euclid’s lemma, the Fibonacci sequence, and unique factorization Coverage of important theorems including the well-ordering theorem, completeness theorem, compactness theorem, as well as the theorems of Löwenheim–Skolem, Burali-Forti, Hartogs, Cantor–Schröder–Bernstein, and König An excellent textbook for students studying the foundations of mathematics and mathematical proofs, A First Course in Mathematical Logic and Set Theory is also appropriate for readers preparing for careers in mathematics education or computer science. In addition, the book is ideal for introductory courses on mathematical logic and/or set theory and appropriate for upper-undergraduate transition courses with rigorous mathematical reasoning involving algebra, number theory, or analysis.

This introduction to mathematical logic explores philosophical issues and Gödel's Theorem. Its widespread influence extends to the author of Gödel, Escher, Bach, whose Pulitzer Prize–winning book was inspired by this work.

Peter Smith examines Gödel's Theorems, how they were established and why they matter.

While there are already several well known textbooks on mathematical logic this book is unique in treating the material in a concise and streamlined fashion. This allows many important topics to be covered in a one semester course. Although the book is intended for use as a graduate text the first three chapters can be understood by undergraduates interested in mathematical logic. The remaining chapters contain material on logic programming for computer scientists, model theory, recursion theory, Godel’s Incompleteness Theorems, and applications of mathematical logic. Philosophical and foundational problems of mathematics are discussed throughout the text.

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This book deals with two important branches of mathematics, namely, logic and set theory. Logic and set theory are closely related and play very crucial roles in the foundation of mathematics, and together produce several results in all of mathematics. The topics of logic and set theory are required in many areas of physical sciences, engineering, and technology. The book offers solved examples and exercises, and provides reasonable details to each topic discussed, for easy understanding. The book is designed for readers from various disciplines where mathematical logic and set theory play a crucial role. The book will be of interested to students and instructors in engineering, mathematics, computer science, and technology.

A serious introductory treatment geared toward non-logicians, this survey traces the development of mathematical logic from ancient to modern times and discusses the work of Planck, Einstein, Bohr, Pauli, Heisenberg, Dirac, and others. 1972 edition.

This book, presented in two parts, offers a slow introduction to mathematical logic, and several basic concepts of model theory, such as first-order definability, types, symmetries, and elementary extensions. Its first part, Logic Sets, and Numbers, shows how mathematical logic is used to develop the number structures of classical mathematics. The exposition does not assume any prerequisites; it is rigorous, but as informal as possible. All necessary concepts are introduced exactly as they would be in a course in mathematical logic; but are accompanied by more extensive introductory remarks and examples to motivate formal developments. The second part, Relations, Structures, Geometry, introduces several basic concepts of model theory, such as first-order definability, types, symmetries, and elementary extensions, and shows how they are used to study and classify mathematical structures. Although more advanced, this second part is accessible to the reader who is either already familiar with basic mathematical logic, or has carefully read the first part of the book. Classical developments in model theory, including the Compactness Theorem and its uses, are discussed. Other topics include tameness, minimality, and order minimality of structures. The book can be used as an introduction to model theory, but unlike standard texts, it does not require familiarity with abstract algebra. This book will also be of interest to mathematicians who know the technical aspects of the subject, but are not familiar with its history and philosophical background.

Algebraic Methods of Mathematical Logic focuses on the algebraic methods of mathematical logic, including Boolean algebra, mathematical language, and arithmetization. The book first offers information on the dialectic of the relation between mathematical and metamathematical aspects; metamathematico-mathematical parallelism and its natural limits; practical applications of methods of mathematical logic; and principal mathematical tools of mathematical logic. The text then elaborates on the language of mathematics and its symbolization and recursive construction of the relation of consequence. Discussions focus on recursive construction of the relation of consequence, fundamental descriptively-semantic rules, mathematical logic and mathematical language as a material system of signs, and the substance and purpose of symbolization of mathematical language. The publication examines expressive possibilities of symbolization; intuitive and mathematical notions of an idealized axiomatic mathematical theory; and the algebraic theory of elementary predicate logic. Topics include the notion of Boolean algebra based on joins, meets, and complementation, logical frame of a language and mathematical theory, and arithmetization and algebraization. The manuscript is a valuable reference for mathematicians and researchers interested in the algebraic methods of mathematical logic.

This textbook introduces discrete mathematics by emphasizing the importance of reading and writing proofs. Because it begins by carefully establishing a familiarity with mathematical logic and proof, this approach suits not only a discrete mathematics course, but can also function as a transition to proof. Its unique, deductive perspective on mathematical logic provides students with the tools to more deeply understand mathematical methodology—an approach that the author has successfully classroom tested for decades. Chapters are helpfully organized so that, as they escalate in complexity, their underlying connections are easily identifiable. Mathematical logic and proofs are first introduced before moving onto more complex topics in discrete mathematics. Some of these topics include: Mathematical and structural induction Set theory Combinatorics Functions, relations, and ordered sets Boolean algebra and Boolean functions Graph theory Introduction to Discrete Mathematics via Logic and Proof will suit intermediate undergraduates majoring in mathematics, computer science, engineering, and related subjects with no formal prerequisites beyond a background in secondary mathematics.

This is an introductory undergraduate textbook in set theory. In mathematics these days, essentially everything is a set. Some knowledge of set theory is necessary part of the background everyone needs for further study of mathematics. It is also possible to study set theory for its own interest--it is a subject with intruiging results anout simple objects. This book starts with material that nobody can do without. There is no end to what can be learned of set theory, but here is a beginning.

Except for this preface, this study is completely self-contained. It is intended to serve both as an introduction to Quantification Theory and as an exposition of new results and techniques in "analytic" or "cut-free" methods. We use the term "analytic" to apply to any proof procedure which obeys the subformula principle (we think of such a procedure as "analysing" the formula into its successive components). Gentzen cut-free systems are perhaps the best known example of ana lytic proof procedures. Natural deduction systems, though not usually analytic, can be made so (as we demonstrated in [3]). In this study, we emphasize the tableau point of view, since we are struck by its simplicity and mathematical elegance. Chapter I is completely introductory. We begin with preliminary material on trees (necessary for the tableau method), and then treat the basic syntactic and semantic fundamentals of propositional logic. We use the term "Boolean valuation" to mean any assignment of truth values to all formulas which satisfies the usual truth-table conditions for the logical connectives. Given an assignment of truth-values to all propositional variables, the truth-values of all other formulas under this assignment is usually defined by an inductive procedure. We indicate in Chapter I how this inductive definition can be made explicit-to this end we find useful the notion of a formation tree (which we discuss earlier).

This is a systematic and well-paced introduction to mathematical logic. Excellent as a course text, the book does not presuppose any previous knowledge and can be used also for self-study by more ambitious students. Starting with the basics of set theory, induction and computability, it covers propositional and first-order logic their syntax, reasoning systems and semantics. Soundness and completeness results for Hilbert's and Gentzen's systems are presented, along with simple decidability arguments. The general applicability of various concepts and techniques is demonstrated by highlighting their consistent reuse in different contexts. Unlike in most comparable texts, presentation of syntactic reasoning systems precedes the semantic explanations. The simplicity of syntactic constructions and rules of a high, though often neglected, pedagogical value aids students in approaching more complex semantic issues. This order of presentation also brings forth the relative independence of syntax from the semantics, helping to appreciate the importance of the purely symbolic systems, like those underlying computers. An overview of the history of logic precedes the main text, in which careful presentation of concepts, results and examples is accompanied by the informal analogies and illustrations. These informal aspects are kept clearly apart from the technical ones. Together, they form a unique text which may be appreciated equally by lecturers and students occupied with mathematical precision, as well as those interested in the relations of logical formalisms to the problems of computability and the philosophy of mathematical logic.