A First Course In Noncommutative Rings 2nd Edition

There is a well-known correspondence between the objects of algebra and geometry: a space gives rise to a function algebra; a vector bundle over the space corresponds to a projective module over this algebra; cohomology can be read off the de Rham complex; and so on. In this book Yuri Manin addresses a variety of instances in which the application of commutative algebra cannot be used to describe geometric objects, emphasizing the recent upsurge of activity in studying noncommutative rings as if they were function rings on "noncommutative spaces." Manin begins by summarizing and giving examples of some of the ideas that led to the new concepts of noncommutative geometry, such as Connes' noncommutative de Rham complex, supergeometry, and quantum groups. He then discusses supersymmetric algebraic curves that arose in connection with superstring theory; examines superhomogeneous spaces, their Schubert cells, and superanalogues of Weyl groups; and provides an introduction to quantum groups. This book is intended for mathematicians and physicists with some background in Lie groups and complex geometry. Originally published in 1991. The Princeton Legacy Library uses the latest print-on-demand technology to again make available previously out-of-print books from the distinguished backlist of Princeton University Press. These editions preserve the original texts of these important books while presenting them in durable paperback and hardcover editions. The goal of the Princeton Legacy Library is to vastly increase access to the rich scholarly heritage found in the thousands of books published by Princeton University Press since its founding in 1905.

Based in large part on the comprehensive "First Course in Ring Theory" by the same author, this book provides a comprehensive set of problems and solutions in ring theory that will serve not only as a teaching aid to instructors using that book, but also for students, who will see how ring theory theorems are applied to solving ring-theoretic problems and how good proofs are written. The author demonstrates that problem-solving is a lively process: in "Comments" following many solutions he discusses what happens if a hypothesis is removed, whether the exercise can be further generalized, what would be a concrete example for the exercise, and so forth. The book is thus much more than a solution manual.

Noncommutative Rings provides a cross-section of ideas, techniques, and results that give the reader an idea of that part of algebra which concerns itself with noncommutative rings. In the space of 200 pages, Herstein covers the Jacobson radical, semisimple rings, commutativity theorems, simple algebras, representations of finite groups, polynomial identities, Goldie's theorem, and the Golod–Shafarevitch theorem. Almost every practicing ring theorist has studied portions of this classic monograph.

This book is an introduction to module theory for the reader who knows something about linear algebra and ring theory. Its main aim is the derivation of the structure theory of modules over Euclidean domains. This theory is applied to obtain the structure of abelian groups and the rational canonical and Jordan normal forms of matrices. The basic facts about rings and modules are given in full generality, so that some further topics can be discussed, including projective modules and the connection between modules and representations of groups. The book is intended to serve as supplementary reading for the third or fourth year undergraduate who is taking a course in module theory. The further topics point the way to some projects that might be attempted in conjunction with a taught course. Contents:Rings and IdealsEuclidean DomainsModules and SubmodulesHomomorphismsFree ModulesQuotient Modules and Cyclic ModulesDirect Sums of ModulesTorsion and the Primary DecompositionPresentationsDiagonalizing and Inverting MatricesFitting IdealsThe Decomposition of ModulesNormal Forms for MatricesProjective Modules Readership: Final year undergraduates and new graduate students in pure mathematics. Keywords:Module;Commutative Ring;Euclidean Domain;Fitting Ideal;Matrix Diagonalization;Invariant Factor;Elementary Divisor;Rational Canonical Form;Jordan Normal Form

This book on modern module and non-commutative ring theory is ideal for beginning graduate students. It starts at the foundations of the subject and progresses rapidly through the basic concepts to help the reader reach current research frontiers. Students will have the chance to develop proofs, solve problems, and to find interesting questions. The first half of the book is concerned with free, projective, and injective modules, tensor algebras, simple modules and primitive rings, the Jacobson radical, and subdirect products. Later in the book, more advanced topics, such as hereditary rings, categories and functors, flat modules, and purity are introduced. These later chapters will also prove a useful reference for researchers in non-commutative ring theory. Enough background material (including detailed proofs) is supplied to give the student a firm grounding in the subject.

This book introduces the notion of an E-semigroup, a generalization of the known concept of E_O-semigroup. These objects are families of endomorphisms of a von Neumann algebra satisfying certain natural algebraic and continuity conditions. Its thorough approach is ideal for graduate students and research mathematicians.

Considered a classic by many, A First Course in Abstract Algebra is an in-depth, introductory text which gives students a firm foundation for more specialized work by emphasizing an understanding of the nature of algebraic structures. The Sixth Edition continues its tradition of teaching in a classical manner, while integrating field theory and new exercises.

This book introduces harmonic analysis at an undergraduate level. In doing so it covers Fourier analysis and paves the way for Poisson Summation Formula. Another central feature is that is makes the reader aware of the fact that both principal incarnations of Fourier theory, the Fourier series and the Fourier transform, are special cases of a more general theory arising in the context of locally compact abelian groups. The final goal of this book is to introduce the reader to the techniques used in harmonic analysis of noncommutative groups. These techniques are explained in the context of matrix groups as a principal example.

Aimed at the novice rather than the connoisseur and stressing the role of examples and motivation, this text is suitable not only for use in a graduate course, but also for self-study in the subject by interested graduate students. More than 400 exercises testing the understanding of the general theory in the text are included in this new edition.

Providing an elementary introduction to noncommutative rings and algebras, this textbook begins with the classical theory of finite dimensional algebras. Only after this, modules, vector spaces over division rings, and tensor products are introduced and studied. This is followed by Jacobson's structure theory of rings. The final chapters treat free algebras, polynomial identities, and rings of quotients. Many of the results are not presented in their full generality. Rather, the emphasis is on clarity of exposition and simplicity of the proofs, with several being different from those in other texts on the subject. Prerequisites are kept to a minimum, and new concepts are introduced gradually and are carefully motivated. Introduction to Noncommutative Algebra is therefore accessible to a wide mathematical audience. It is, however, primarily intended for beginning graduate and advanced undergraduate students encountering noncommutative algebra for the first time.

This volume offers a compendium of exercises of varying degree of difficulty in the theory of modules and rings. It is the companion volume to GTM 189. All exercises are solved in full detail. Each section begins with an introduction giving the general background and the theoretical basis for the problems that follow.

This book introduces the theory of modular forms, from which all rational elliptic curves arise, with an eye toward the Modularity Theorem. Discussion covers elliptic curves as complex tori and as algebraic curves; modular curves as Riemann surfaces and as algebraic curves; Hecke operators and Atkin-Lehner theory; Hecke eigenforms and their arithmetic properties; the Jacobians of modular curves and the Abelian varieties associated to Hecke eigenforms. As it presents these ideas, the book states the Modularity Theorem in various forms, relating them to each other and touching on their applications to number theory. The authors assume no background in algebraic number theory and algebraic geometry. Exercises are included.

Realizing the specific needs of first-year graduate students, this reference allows readers to grasp and master fundamental concepts in abstract algebra-establishing a clear understanding of basic linear algebra and number, group, and commutative ring theory and progressing to sophisticated discussions on Galois and Sylow theory, the structure of abelian groups, the Jordan canonical form, and linear transformations and their matrix representations.

One of my favorite graduate courses at Berkeley is Math 251, a one-semester course in ring theory offered to second-year level graduate students. I taught this course in the Fall of 1983, and more recently in the Spring of 1990, both times focusing on the theory of noncommutative rings. This book is an outgrowth of my lectures in these two courses, and is intended for use by instructors and graduate students in a similar one-semester course in basic ring theory. Ring theory is a subject of central importance in algebra. Historically, some of the major discoveries in ring theory have helped shape the course of development of modern abstract algebra. Today, ring theory is a fer tile meeting ground for group theory (group rings), representation theory (modules), functional analysis (operator algebras), Lie theory (enveloping algebras), algebraic geometry (finitely generated algebras, differential op erators, invariant theory), arithmetic (orders, Brauer groups), universal algebra (varieties of rings), and homological algebra (cohomology of rings, projective modules, Grothendieck and higher K-groups). In view of these basic connections between ring theory and other branches of mathemat ics, it is perhaps no exaggeration to say that a course in ring theory is an indispensable part of the education for any fledgling algebraist. The purpose of my lectures was to give a general introduction to the theory of rings, building on what the students have learned from a stan dard first-year graduate course in abstract algebra.

This introduction to noncommutative noetherian rings is intended to be accessible to anyone with a basic background in abstract algebra. It can be used as a second-year graduate text, or as a self-contained reference. Extensive explanatory discussion is given, and exercises are integrated throughout. This edition incorporates substantial revisions, particularly in the first third of the book, where the presentation has been changed to increase accessibility and topicality. New material includes the basic types of quantum groups, which then serve as test cases for the theory developed.

Considered a classic by many, A First Course in Abstract Algebra is an in-depth introduction to abstract algebra. Focused on groups, rings and fields, this text gives students a firm foundation for more specialized work by emphasizing an understanding of the nature of algebraic structures.

Noncommutative Geometry is one of the most deep and vital research subjects of present-day Mathematics. Its development, mainly due to Alain Connes, is providing an increasing number of applications and deeper insights for instance in Foliations, K-Theory, Index Theory, Number Theory but also in Quantum Physics of elementary particles. The purpose of the Summer School in Martina Franca was to offer a fresh invitation to the subject and closely related topics; the contributions in this volume include the four main lectures, cover advanced developments and are delivered by prominent specialists.

This is a concise 2000 introduction at graduate level to ring theory, module theory and number theory.

Noncommutative geometry, inspired by quantum physics, describes singular spaces by their noncommutative coordinate algebras and metric structures by Dirac-like operators. Such metric geometries are described mathematically by Connes' theory of spectral triples. These lectures, delivered at an EMS Summer School on noncommutative geometry and its applications, provide an overview of spectral triples based on examples. This introduction is aimed at graduate students of both mathematics and theoretical physics. It deals with Dirac operators on spin manifolds, noncommutative tori, Moyal quantization and tangent groupoids, action functionals, and isospectral deformations. The structural framework is the concept of a noncommutative spin geometry; the conditions on spectral triples which determine this concept are developed in detail. The emphasis throughout is on gaining understanding by computing the details of specific examples. The book provides a middle ground between a comprehensive text and a narrowly focused research monograph. It is intended for self-study, enabling the reader to gain access to the essentials of noncommutative geometry. New features since the original course are an expanded bibliography and a survey of more recent examples and applications of spectral triples.

Projective modules: Modules and homomorphisms Projective modules Completely reducible modules Wedderburn rings Artinian rings Hereditary rings Dedekind domains Projective dimension Tensor products Local rings Polynomial rings: Skew polynomial rings Grothendieck groups Graded rings and modules Induced modules Syzygy theorem Patching theorem Serre conjecture Big projectives Generic flatness Nullstellensatz Injective modules: Injective modules Injective dimension Essential extensions Maximal ring of quotients Classical ring of quotients Goldie rings Uniform dimension Uniform injective modules Reduced rank Index

Noncommutative Polynomial Algebras of Solvable Type and Their Modules is the first book to systematically introduce the basic constructive-computational theory and methods developed for investigating solvable polynomial algebras and their modules. In doing so, this book covers: A constructive introduction to solvable polynomial algebras and Gröbner basis theory for left ideals of solvable polynomial algebras and submodules of free modules The new filtered-graded techniques combined with the determination of the existence of graded monomial orderings The elimination theory and methods (for left ideals and submodules of free modules) combining the Gröbner basis techniques with the use of Gelfand-Kirillov dimension, and the construction of different kinds of elimination orderings The computational construction of finite free resolutions (including computation of syzygies, construction of different kinds of finite minimal free resolutions based on computation of different kinds of minimal generating sets), etc. This book is perfectly suited to researchers and postgraduates researching noncommutative computational algebra and would also be an ideal resource for teaching an advanced lecture course.

The volume is based on a course, ``Geometric Models for Noncommutative Algebras'' taught by Professor Weinstein at Berkeley. Noncommutative geometry is the study of noncommutative algebras as if they were algebras of functions on spaces, for example, the commutative algebras associated to affine algebraic varieties, differentiable manifolds, topological spaces, and measure spaces. In this work, the authors discuss several types of geometric objects (in the usual sense of sets with structure) that are closely related to noncommutative algebras. Central to the discussion are symplectic and Poisson manifolds, which arise when noncommutative algebras are obtained by deforming commutative algebras. The authors also give a detailed study of groupoids (whose role in noncommutative geometry has been stressed by Connes) as well as of Lie algebroids, the infinitesimal approximations to differentiable groupoids. Featured are many interesting examples, applications, and exercises. The book starts with basic definitions and builds to (still) open questions. It is suitable for use as a graduate text. An extensive bibliography and index are included.

as a student." --Book Jacket.

This is the first existing volume that collects lectures on this important and fast developing subject in mathematics. The lectures are given by leading experts in the field and the range of topics is kept as broad as possible by including both the algebraic and the differential aspects of noncommutative geometry as well as recent applications to theoretical physics and number theory.

About This Book This book is meant to be used by beginning graduate students. It covers basic material needed by any student of algebra, and is essential to those specializing in ring theory, homological algebra, representation theory and K-theory, among others. It will also be of interest to students of algebraic topology, functional analysis, differential geometry and number theory. Our approach is more homological than ring-theoretic, as this leads the to many important areas of mathematics. This ap student more quickly proach is also, we believe, cleaner and easier to understand. However, the more classical, ring-theoretic approach, as well as modern extensions, are also presented via several exercises and sections in Chapter Five. We have tried not to leave any gaps on the paths to proving the main theorem- at most we ask the reader to fill in details for some of the sideline results; indeed this can be a fruitful way of solidifying one's understanding.

This book provides an introduction to noncommutative geometry and presents a number of its recent applications to particle physics. It is intended for graduate students in mathematics/theoretical physics who are new to the field of noncommutative geometry, as well as for researchers in mathematics/theoretical physics with an interest in the physical applications of noncommutative geometry. In the first part, we introduce the main concepts and techniques by studying finite noncommutative spaces, providing a “light” approach to noncommutative geometry. We then proceed with the general framework by defining and analyzing noncommutative spin manifolds and deriving some main results on them, such as the local index formula. In the second part, we show how noncommutative spin manifolds naturally give rise to gauge theories, applying this principle to specific examples. We subsequently geometrically derive abelian and non-abelian Yang-Mills gauge theories, and eventually the full Standard Model of particle physics, and conclude by explaining how noncommutative geometry might indicate how to proceed beyond the Standard Model.

The unifying theme of this book is the interplay among noncommutative geometry, physics, and number theory. The two main objects of investigation are spaces where both the noncommutative and the motivic aspects come to play a role: space-time, where the guiding principle is the problem of developing a quantum theory of gravity, and the space of primes, where one can regard the Riemann Hypothesis as a long-standing problem motivating the development of new geometric tools. The book stresses the relevance of noncommutative geometry in dealing with these two spaces. The first part of the book deals with quantum field theory and the geometric structure of renormalization as a Riemann-Hilbert correspondence. It also presents a model of elementary particle physics based on noncommutative geometry. The main result is a complete derivation of the full Standard Model Lagrangian from a very simple mathematical input. Other topics covered in the first part of the book are a noncommutative geometry model of dimensional regularization and its role in anomaly computations, and a brief introduction to motives and their conjectural relation to quantum field theory. The second part of the book gives an interpretation of the Weil explicit formula as a trace formula and a spectral realization of the zeros of the Riemann zeta function. This is based on the noncommutative geometry of the adèle class space, which is also described as the space of commensurability classes of Q-lattices, and is dual to a noncommutative motive (endomotive) whose cyclic homology provides a general setting for spectral realizations of zeros of L-functions. The quantum statistical mechanics of the space of Q-lattices, in one and two dimensions, exhibits spontaneous symmetry breaking. In the low-temperature regime, the equilibrium states of the corresponding systems are related to points of classical moduli spaces and the symmetries to the class field theory of the field of rational numbers and of imaginary quadratic fields, as well as to the automorphisms of the field of modular functions. The book ends with a set of analogies between the noncommutative geometries underlying the mathematical formulation of the Standard Model minimally coupled to gravity and the moduli spaces of Q-lattices used in the study of the zeta function.

"Basic Noncommutative Geometry provides an introduction to noncommutative geometry and some of its applications. The book can be used either as a textbook for a graduate course on the subject or for self-study. It will be useful for graduate students and researchers in mathematics and theoretical physics and all those who are interested in gaining an understanding of the subject. One feature of this book is the wealth of examples and exercises that help the reader to navigate through the subject. While background material is provided in the text and in several appendices, some familiarity with basic notions of functional analysis, algebraic topology, differential geometry and homological algebra at a first year graduate level is helpful. Developed by Alain Connes since the late 1970s, noncommutative geometry has found many applications to long-standing conjectures in topology and geometry and has recently made headways in theoretical physics and number theory. The book starts with a detailed description of some of the most pertinent algebra-geometry correspondences by casting geometric notions in algebraic terms, then proceeds in the second chapter to the idea of a noncommutative space and how it is constructed. The last two chapters deal with homological tools: cyclic cohomology and Connes-

Chern characters in K-theory and K-homology, culminating in one commutative diagram expressing the equality of topological and analytic index in a noncommutative setting. Applications to integrality of noncommutative topological invariants are given as well."--Publisher's description.

The Book of R is a comprehensive, beginner-friendly guide to R, the world’s most popular programming language for statistical analysis. Even if you have no programming experience and little more than a grounding in the basics of mathematics, you’ll find everything you need to begin using R effectively for statistical analysis. You’ll start with the basics, like how to handle data and write simple programs, before moving on to more advanced topics, like producing statistical summaries of your data and performing statistical tests and modeling. You’ll even learn how to create impressive data visualizations with R’s basic graphics tools and contributed packages, like ggplot2 and ggvis, as well as interactive 3D visualizations using the rgl package. Dozens of hands-on exercises (with downloadable solutions) take you from theory to practice, as you learn: –The fundamentals of programming in R, including how to write data frames, create functions, and use variables, statements, and loops –Statistical concepts like exploratory data analysis, probabilities, hypothesis tests, and regression modeling, and how to execute them in R –How to access R’s thousands of functions, libraries, and data sets –How to draw valid and useful conclusions from your data –How to create publication-quality graphics of your results Combining detailed explanations with real-world examples and exercises, this book will provide you with a solid understanding of both statistics and the depth of R’s functionality. Make The Book of R your doorway into the growing world of data analysis.

This graduate-level text provides a thorough grounding in the representation theory of finite groups over fields and rings. The book provides a balanced and comprehensive account of the subject, detailing the methods needed to analyze representations that arise in many areas of mathematics. Key topics include the construction and use of character tables, the role of induction and restriction, projective and simple modules for group algebras, indecomposable representations, Brauer characters, and block theory. This classroom-tested text provides motivation through a large number of worked examples, with exercises at the end of each chapter that test the reader's knowledge, provide further examples and practice, and include results not proven in the text. Prerequisites include a graduate course in abstract algebra, and familiarity with the properties of groups, rings, field extensions, and linear algebra.

This book is an expanded text for a graduate course in commutative algebra, focusing on the algebraic underpinnings of algebraic geometry and of number theory. Accordingly, the theory of affine algebras is featured, treated both directly and via the theory of Noetherian and Artinian modules, and the theory of graded algebras is included to provide the foundation for projective varieties. Major topics include the theory of modules over a principal ideal domain, and its applicationsto matrix theory (including the Jordan decomposition), the Galois theory of field extensions, transcendence degree, the prime spectrum of an algebra, localization, and the classical theory of Noetherian and Artinian rings. Later chapters include some algebraic theory of elliptic curves (featuring theMordell-Weil theorem) and valuation theory, including local fields. One feature of the book is an extension of the text through a series of appendices. This permits the inclusion of more advanced material, such as transcendental field extensions, the discriminant and resultant, the theory of Dedekind domains, and basic theorems of rings of algebraic integers. An extended appendix on derivations includes the Jacobian conjecture and Makar-Limanov's theory of locally nilpotent derivations. Grobnerbases can be found in another appendix. Exercises provide a further extension of the text. The book can be used both as a textbook and as a reference source.

A First Course in Noncommutative RingsSpringer Science & Business Media

This new book can be read independently from the first volume and may be used for lecturing, seminar- and self-study, or for general reference. It focuses more on specific topics in order to introduce readers to a wealth of basic and useful ideas without the hindrance of heavy machinery or undue abstractions. User-friendly with its abundance of examples illustrating the theory at virtually every step, the volume contains a large number of carefully chosen exercises to provide newcomers with practice, while offering a rich additional source of information to experts. A direct approach is used in order to present the material in an efficient and economic way, thereby introducing readers to a considerable amount of interesting ring theory without being dragged through endless preparatory material.

A polynomial identity for an algebra (or a ring) A A is a polynomial in noncommutative variables that vanishes under any evaluation in A A. An algebra satisfying a nontrivial polynomial identity is called a PI algebra, and this is the main object of study in this book, which can be used by graduate students and researchers alike. The book is divided into four parts. Part 1 contains foundational material on representation theory and noncommutative algebra. In addition to setting the stage for the rest of the book, this part can be used for an introductory course in noncommutative algebra. An expert reader may use Part 1 as reference and start with the main topics in the remaining parts. Part 2 discusses the combinatorial aspects of the theory, the growth theorem, and Shirshov's bases. Here methods of representation theory of the symmetric group play a major role. Part 3 contains the main body of structure theorems for PI algebras, theorems of Kaplansky and Posner, the theory of central polynomials, M. Artin's theorem on Azumaya algebras, and the geometric part on the variety of semisimple representations, including the foundations of the theory of Cayley–Hamilton algebras. Part 4 is devoted first to the proof of the theorem of Razmyslov, Kemer, and Braun on the nilpotency of the nil radical for finitely generated PI algebras over Noetherian rings, then to the theory of Kemer and the Specht problem. Finally, the authors discuss PI exponent and codimension growth. This part uses some nontrivial analytic tools coming from probability theory. The appendix presents the counterexamples of Golod and Shafarevich to the Burnside problem.

A modern account of the subject and its applications. Excellent resource for those working in algebra and theoretical computer science.

This book is a collection of invited papers and articles, many presented at the 2008 International Conference on Ring and Module Theory. The papers explore the latest in various areas of algebra, including ring theory, module theory and commutative algebra.

This work offers a comprehensive account of skew fields and related mathematics.